

Double-lepton polarization asymmetries in the Exclusive $B \rightarrow \rho \ell^+ \ell^-$ decay beyond the Standard Model

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Abstract

The double-lepton polarization asymmetries in $B \rightarrow \rho \ell^+ \ell^-$ decay is analyzed in a model independent framework. The general expressions for nine double-polarization asymmetries are calculated. It is shown that the study of the double-lepton polarization asymmetries proves to be very useful tool in looking for new physics beyond the standard model.

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1 Introduction

Rare B meson decays, induced by flavor changing neutral current (FCNC) $b \rightarrow s(d)\ell^+\ell^-$ transitions constitute one of the most important class of decays for testing the gauge structure of the Standard Model (SM). These decays which are forbidden in the SM at tree level, occur at loop level and provide insight to check the predictions of the SM at quantum level. Moreover, these decays are also quite sensitive to the existence of new physics beyond the SM, since new particles running at loops can give contribution to these decays. The new physics manifests itself in rare decays in two different ways; one via modification of the existing Wilson coefficients in the SM, or through the introduction of some new operators with new coefficients which are absent in the SM. Some of the most important exclusive FCNC decays governed by $b \rightarrow s(d)$ transition at quark level are $B \rightarrow K^*\gamma$ and $B \rightarrow (\pi, \rho, K, K^*)\ell^+\ell^-$ decays. The decays of the kind $B \rightarrow M\ell^+\ell^-$, where M stands for pseudoscalar or vector mesons, enable the investigation of the experimental observables, such as, lepton pair forward-backward (FB) asymmetry, lepton polarizations, etc. One of the most efficient ways in looking for new physics beyond the SM is the measurement of lepton polarization in the decays. Polarization of a single lepton has been studied in $B \rightarrow K^*\ell^+\ell^-$ [1], $B \rightarrow X_s\ell^+\ell^-$ [2, 3], $B \rightarrow K\ell^+\ell^-$ [4], $B \rightarrow \pi(\rho)\ell^+\ell^-$ [5, 6] and $B \rightarrow \ell^+\ell^-\gamma$ [7] decays in detail in fitting the parameters of the SM and set constraints on new physics beyond the SM. Moreover, as has already been pointed out in [8], some of the single lepton polarization asymmetries might be quite small to be observed and might not provide sufficient number of observables in checking the structure of the effective Hamiltonian. By taking both lepton polarizations into account simultaneously, maximum number of independent observables are constructed. It is clear that, measurement of many more observables which would be useful in further improvement of the parameters of the SM probing new physics beyond the SM. It should be noted here that both lepton polarizations in the $B \rightarrow K^*\tau^+\tau^-$ and $B \rightarrow K\ell^+\ell^-$ decays are studied in [9] and [10], respectively. The decays of B mesons induced by the $b \rightarrow d\ell^+\ell^-$ transition are promising in looking for CP violation since the CKM factors $V_{tb}V_{td}^*$, $V_{ub}V_{ud}^*$ and $V_{cb}V_{cd}^*$ in the SM are all of the same order. For this reason CP violation is much more considerable in the decays induced by $b \rightarrow d$ transition. So, study of the exclusive decays $B_d \rightarrow (\pi, \rho, \eta)\ell^+\ell^-$ are quite promising for the confirmation of the CP violation and these decays have extensively been investigated in the SM [11] and beyond [12].

The aim of the present work is to study the double-lepton polarization asymmetries in the exclusive $B \rightarrow \rho\ell^+\ell^-$ decay in a model independent way, including all possible forms of interactions into the effective Hamiltonian. Moreover, we study the correlation between the double-lepton polarization asymmetries and the branching ratio of the $B \rightarrow \rho\ell^+\ell^-$ decay, in order to find such regions of new Wilson coefficients in which the branching ratio (as well as single-lepton polarization) coincides with the SM prediction while the double-lepton polarization asymmetries do not. It is clear that if such a region of the new Wilson coefficients exists it is an indication of the fact that new physics beyond the SM can be established by measurement of the double-lepton polarizations only. Note that the double-lepton polarizations in the $B \rightarrow K\ell^+\ell^-$ and $B \rightarrow \ell^+\ell^-\gamma$ decays are studied in [13] and [14] in detail.

The paper is organized as follows. In section 2, using a general form of the effective

Hamiltonian, we obtain the matrix element in terms of the form factors of the $B \rightarrow \rho$ transition. In section 3 we derive the analytical results for the nine double-lepton polarization asymmetries. Last section is devoted to the numerical analysis, discussion and conclusions.

2 Double lepton polarization asymmetries in $B \rightarrow \rho \ell^+ \ell^-$ decay

In this section we calculate the double lepton polarizations using a general form of the effective Hamiltonian. The $B \rightarrow \rho \ell^+ \ell^-$ process is governed by $b \rightarrow d \ell^+ \ell^-$ transition at quark level. The matrix element for the $b \rightarrow d \ell^+ \ell^-$ transition can be written in terms of the twelve model independent four-Fermi interactions in the following form:

$$\begin{aligned} \mathcal{H}_{eff} = & \frac{G_F \alpha}{\sqrt{2} \pi} V_{td} V_{tb}^* \left\{ C_{SL} \bar{d}_R i \sigma_{\mu\nu} \frac{q^\nu}{q^2} b_L \bar{\ell} \gamma^\mu \ell + C_{BR} \bar{d}_L i \sigma_{\mu\nu} \frac{q^\nu}{q^2} b_R \bar{\ell} \gamma^\mu \ell \right. \\ & + C_{LL}^{tot} \bar{d}_L \gamma_\mu b_L \bar{\ell}_L \gamma^\mu \ell_L + C_{LR}^{tot} \bar{d}_L \gamma_\mu b_L \bar{\ell}_R \gamma^\mu \ell_R + C_{RL} \bar{d}_R \gamma_\mu b_R \bar{\ell}_L \gamma^\mu \ell_L \\ & + C_{RR} \bar{d}_R \gamma_\mu b_R \bar{\ell}_R \gamma^\mu \ell_R + C_{LRLR} \bar{d}_L b_R \bar{\ell}_L \ell_R + C_{RLLR} \bar{d}_R b_L \bar{\ell}_L \ell_R \\ & + C_{LRRL} \bar{d}_L b_R \bar{\ell}_R \ell_L + C_{RLRL} \bar{d}_R b_L \bar{\ell}_R \ell_L + C_T \bar{d} \sigma_{\mu\nu} b \bar{\ell} \sigma^{\mu\nu} \ell \\ & \left. + i C_{TE} \epsilon^{\mu\nu\alpha\beta} \bar{d} \sigma_{\mu\nu} b \bar{\ell} \sigma_{\alpha\beta} \ell \right\}, \end{aligned} \quad (1)$$

where

$$d_L = \frac{1 - \gamma_5}{2} d, \quad d_R = \frac{1 + \gamma_5}{2} d,$$

C_X are the coefficients of the four-Fermi interactions and q is the momentum transfer. Among all these Wilson coefficients, several already exists in the SM. Indeed, the first two coefficients in Eq. (1), C_{SL} and C_{BR} , are the nonlocal Fermi interactions, which correspond to $-2m_s C_7^{eff}$ and $-2m_b C_7^{eff}$ in the SM, respectively. The next four terms with coefficients C_{LL} , C_{LR} , C_{RL} and C_{RR} are the vector type interactions. Two of these vector interactions containing C_{LL}^{tot} and C_{LR}^{tot} do already exist in the SM in the form $(C_9^{eff} - C_{10})$ and $(C_9^{eff} + C_{10})$. Therefore, C_{LL}^{tot} and C_{LR}^{tot} can be written as

$$\begin{aligned} C_{LL}^{tot} &= C_9^{eff} - C_{10} + C_{LL}, \\ C_{LR}^{tot} &= C_9^{eff} + C_{10} + C_{LR}, \end{aligned}$$

where C_{LL} and C_{LR} describe the contributions of the new physics. The terms with coefficients C_{LRLR} , C_{RLLR} , C_{LRRL} and C_{RLRL} describe the scalar type interactions. The remaining last two terms lead by the coefficients C_T and C_{TE} , obviously, describe the tensor type interactions.

It should be noted here that, in further analysis we will assume that all new Wilson coefficients are real, as is the case in the SM, while only C_9^{eff} contains imaginary part and it is parametrized in the following form

$$C_9^{eff} = \xi_1 + \lambda_u \xi_2, \quad (2)$$

where

$$\lambda_u = \frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*} ,$$

and

$$\begin{aligned} \xi_1 &= 4.128 + 0.138\omega(\hat{s}) + g(\hat{m}_c, \hat{s})C_0(\hat{m}_b) - \frac{1}{2}g(\hat{m}_d, \hat{s})(C_3 + C_4) \\ &\quad - \frac{1}{2}g(\hat{m}_b, \hat{s})(4C_3 + 4C_4 + 3C_5 + C_6) + \frac{2}{9}(3C_3 + C_4 + 3C_5 + C_6) , \\ \xi_2 &= [g(\hat{m}_c, \hat{s}) - g(\hat{m}_u, \hat{s})](3C_1 + C_2) , \end{aligned} \quad (3)$$

where $\hat{m}_q = m_q/m_b$, $\hat{s} = q^2$, $C_0(\mu) = 3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6$, and

$$\begin{aligned} \omega(\hat{s}) &= -\frac{2}{9}\pi^2 - \frac{4}{3}Li_2(\hat{s}) - \frac{2}{3}\ln(\hat{s})\ln(1-\hat{s}) - \frac{5+4\hat{s}}{3(1+2\hat{s})}\ln(1-\hat{s}) \\ &\quad - \frac{2\hat{s}(1+\hat{s})(1-2\hat{s})}{3(1-\hat{s})^2(1+2\hat{s})}\ln(\hat{s}) + \frac{5+9\hat{s}-6\hat{s}^2}{3(1-\hat{s})(1+2\hat{s})} , \end{aligned} \quad (4)$$

represents the $O(\alpha_s)$ correction coming from one gluon exchange in the matrix element of the operator \mathcal{O}_9 [15], while the function $g(\hat{m}_q, \hat{s})$ represents one-loop corrections to the four-quark operators O_1 – O_6 [16], whose form is

$$\begin{aligned} g(\hat{m}_q, \hat{s}) &= -\frac{8}{9}\ln(\hat{m}_q) + \frac{8}{27} + \frac{4}{9}y_q - \frac{2}{9}(2+y_q) \\ &\quad - \sqrt{|1-y_q|} \left\{ \theta(1-y_q) \left[\ln \left(\frac{1+\sqrt{1-y_q}}{1-\sqrt{1-y_q}} \right) - i\pi \right] + \theta(y_q-1) \arctan \left(\frac{1}{\sqrt{y_q-1}} \right) \right\} , \end{aligned} \quad (5)$$

where $y_q = 4\hat{m}_q^2/\hat{s}$.

In addition to the short distance contributions, $B \rightarrow X_d \ell^+ \ell^-$ decay also receives long distance contributions, which have their origin in the real $\bar{u}u$, $\bar{d}d$ and $\bar{c}c$ intermediate states, i.e., ρ , ω and J/ψ family. There are four different approaches in taking long distance contributions into consideration: a) HQET based approach [17], b) AMM approach [18], c) LSW approach [19], and d) KS approach [20]. In the present work we choose the AMM approach, in which these resonance contributions are parametrized using the Breit–Wigner form for the resonant states. The effective coefficient C_9^{eff} including the ρ , ω and J/ψ resonances are defined as

$$C_9^{eff} = C_9(\mu) + Y_{res}(\hat{s}) , \quad (6)$$

where

$$\begin{aligned} Y_{res} &= -\frac{3\pi}{\alpha^2} \left\{ \left(C^{(0)}(\mu) + \lambda_u [3C_1(\mu) + C_2(\mu)] \right) \sum_{V_i=\psi} K_i \frac{\Gamma(V_i \rightarrow \ell^+ \ell^-) M_{V_i}}{M_{V_i}^2 - q^2 - iM_{V_i}\Gamma_{V_i}} \right. \\ &\quad \left. - \lambda_u g(\hat{m}_u, \hat{s}) [3C_1(\mu) + C_2(\mu)] \sum_{V_i=\rho, \omega} \frac{\Gamma(V_i \rightarrow \ell^+ \ell^-) M_{V_i}}{M_{V_i}^2 - q^2 - iM_{V_i}\Gamma_{V_i}} \right\} . \end{aligned} \quad (7)$$

The phenomenological factor K_i has the universal value for the inclusive $B \rightarrow X_{s(d)} \ell^+ \ell^-$ decay $K_i \simeq 2.3$ [21], which we use in our calculations.

The decay amplitude for the exclusive $B \rightarrow \rho \ell^+ \ell^-$ decay is obtained from the matrix element of the effective Hamiltonian in Eq. (1) over B and ρ meson states, which can be parametrized in terms of various form factors. It follows from (1) that, the following matrix elements

$$\begin{aligned} & \langle \rho | \bar{d} \gamma_\mu (1 \pm \gamma_5) b | B \rangle , \\ & \langle \rho | \bar{d} i \sigma_{\mu\nu} q^\nu (1 \pm \gamma_5) b | B \rangle , \\ & \langle \rho | \bar{d} (1 \pm \gamma_5) b | B \rangle , \\ & \langle \rho | \bar{d} \sigma_{\mu\nu} b | B \rangle , \end{aligned}$$

are needed in obtaining the decay amplitude of the $B \rightarrow \rho \ell^+ \ell^-$ decay. These matrix elements are defined as follows:

$$\begin{aligned} & \langle \rho(p_\rho, \varepsilon) | \bar{d} \gamma_\mu (1 \pm \gamma_5) b | B(p_B) \rangle = \\ & -\epsilon_{\mu\nu\lambda\sigma} \varepsilon^{*\nu} p_\rho^\lambda q^\sigma \frac{2V(q^2)}{m_B + m_\rho} \pm i \varepsilon_\mu^* (m_B + m_\rho) A_1(q^2) \end{aligned} \quad (8)$$

$$\begin{aligned} & \langle \rho(p_\rho, \varepsilon) | \bar{d} i \sigma_{\mu\nu} q^\nu (1 \pm \gamma_5) b | B(p_B) \rangle = \\ & 4\epsilon_{\mu\nu\lambda\sigma} \varepsilon^{*\nu} p_\rho^\lambda q^\sigma T_1(q^2) \pm 2i \left[\varepsilon_\mu^* (m_B^2 - m_\rho^2) - (p_B + p_\rho)_\mu (\varepsilon^* q) \right] T_2(q^2) \\ & \pm 2i (\varepsilon^* q) \left[q_\mu - (p_B + p_\rho)_\mu \frac{q^2}{m_B^2 - m_\rho^2} \right] T_3(q^2) , \end{aligned} \quad (9)$$

$$\begin{aligned} & \langle \rho(p_\rho, \varepsilon) | \bar{d} \sigma_{\mu\nu} b | B(p_B) \rangle = \\ & i \epsilon_{\mu\nu\lambda\sigma} \left\{ -2T_1(q^2) \varepsilon^{*\lambda} (p_B + p_\rho)^\sigma + \frac{2}{q^2} (m_B^2 - m_\rho^2) [T_1(q^2) - T_2(q^2)] \varepsilon^{*\lambda} q^\sigma \right. \\ & \left. - \frac{4}{q^2} \left[T_1(q^2) - T_2(q^2) - \frac{q^2}{m_B^2 - m_\rho^2} T_3(q^2) \right] (\varepsilon^* q) p_\rho^\lambda q^\sigma \right\} . \end{aligned} \quad (10)$$

where $q = p_B - p_\rho$ is the momentum transfer and ε is the polarization vector of ρ meson.

Note that the matrix element

$$\langle \rho(p_\rho, \varepsilon) | \bar{d} \sigma_{\mu\nu} \gamma_5 b | B(p_B) \rangle$$

can easily be obtained from (10) by using the identity

$$\sigma_{\alpha\beta} = -\frac{i}{2} \epsilon_{\alpha\beta\rho\sigma} \sigma^{\rho\sigma} \gamma_5 .$$

In order to ensure finiteness of (8) and (10) at $q^2 = 0$, we assume that $A_3(q^2 = 0) = A_0(q^2 = 0)$ and $T_1(q^2 = 0) = T_2(q^2 = 0)$. The matrix element $\langle \rho | \bar{d} (1 \pm \gamma_5) b | B \rangle$ can be calculated

by contracting both sides of Eq. (8) with q^μ and using equation of motion. Neglecting the mass of the d quark we get

$$\langle \rho(p_\rho, \varepsilon) | \bar{d}(1 \pm \gamma_5)b | B(p_B) \rangle = \frac{1}{m_b} \left[\mp 2im_\rho(\varepsilon^* q) A_0(q^2) \right]. \quad (11)$$

In deriving Eq. (11) we have used the relationship

$$2m_\rho A_3(q^2) = (m_B + m_\rho) A_1(q^2) - (m_B - m_\rho) A_2(q^2),$$

which follows from the equations of motion.

Using the definition of the form factors, as given above, the amplitude of the $B \rightarrow \rho \ell^+ \ell^-$ decay can be written as

$$\begin{aligned} \mathcal{M}(B \rightarrow \rho \ell^+ \ell^-) &= \frac{G\alpha}{4\sqrt{2}\pi} V_{tb} V_{td}^* \\ &\times \left\{ \bar{\ell} \gamma^\mu (1 - \gamma_5) \ell \left[-2A_1 \epsilon_{\mu\nu\lambda\sigma} \varepsilon^{*\nu} p_\rho^\lambda q^\sigma - iB_1 \varepsilon_\mu^* + iB_2(\varepsilon^* q)(p_B + p_\rho)_\mu + iB_3(\varepsilon^* q) q_\mu \right] \right. \\ &+ \bar{\ell} \gamma^\mu (1 + \gamma_5) \ell \left[-2C_1 \epsilon_{\mu\nu\lambda\sigma} \varepsilon^{*\nu} p_\rho^\lambda q^\sigma - iD_1 \varepsilon_\mu^* + iD_2(\varepsilon^* q)(p_B + p_\rho)_\mu + iD_3(\varepsilon^* q) q_\mu \right] \\ &+ \bar{\ell} (1 - \gamma_5) \ell \left[iB_4(\varepsilon^* q) \right] + \bar{\ell} (1 + \gamma_5) \ell \left[iB_5(\varepsilon^* q) \right] \\ &+ 4\bar{\ell} \sigma^{\mu\nu} \ell \left(iC_T \epsilon_{\mu\nu\lambda\sigma} \right) \left[-2T_1 \varepsilon^{*\lambda} (p_B + p_\rho)^\sigma + B_6 \varepsilon^{*\lambda} q^\sigma - B_7(\varepsilon^* q) p_\rho^\lambda q^\sigma \right] \\ &\left. + 16C_{TE} \bar{\ell} \sigma_{\mu\nu} \ell \left[-2T_1 \varepsilon^{*\mu} (p_B + p_\rho)^\nu + B_6 \varepsilon^{*\mu} q^\nu - B_7(\varepsilon^* q) p_\rho^\mu q^\nu \right] \right\}, \quad (12) \end{aligned}$$

where

$$\begin{aligned} A_1 &= (C_{LL}^{tot} + C_{RL}) \frac{V}{m_B + m_\rho} - 2(C_{BR} + C_{SL}) \frac{T_1}{q^2}, \\ B_1 &= (C_{LL}^{tot} - C_{RL})(m_B + m_\rho) A_1 - 2(C_{BR} - C_{SL})(m_B^2 - m_\rho^2) \frac{T_2}{q^2}, \\ B_2 &= \frac{C_{LL}^{tot} - C_{RL}}{m_B + m_\rho} A_2 - 2(C_{BR} - C_{SL}) \frac{1}{q^2} \left[T_2 + \frac{q^2}{m_B^2 - m_\rho^2} T_3 \right], \\ B_3 &= 2(C_{LL}^{tot} - C_{RL}) m_\rho \frac{A_3 - A_0}{q^2} + 2(C_{BR} - C_{SL}) \frac{T_3}{q^2}, \\ C_1 &= A_1 (C_{LL}^{tot} \rightarrow C_{LR}^{tot}, \quad C_{RL} \rightarrow C_{RR}), \\ D_1 &= B_1 (C_{LL}^{tot} \rightarrow C_{LR}^{tot}, \quad C_{RL} \rightarrow C_{RR}), \\ D_2 &= B_2 (C_{LL}^{tot} \rightarrow C_{LR}^{tot}, \quad C_{RL} \rightarrow C_{RR}), \\ D_3 &= B_3 (C_{LL}^{tot} \rightarrow C_{LR}^{tot}, \quad C_{RL} \rightarrow C_{RR}), \\ B_4 &= -2(C_{LRRL} - C_{RLRL}) \frac{m_\rho}{m_b} A_0, \\ B_5 &= -2(C_{LRRL} - C_{RLLR}) \frac{m_\rho}{m_b} A_0, \\ B_6 &= 2(m_B^2 - m_\rho^2) \frac{T_1 - T_2}{q^2}, \\ B_7 &= \frac{4}{q^2} \left(T_1 - T_2 - \frac{q^2}{m_B^2 - m_\rho^2} T_3 \right). \quad (13) \end{aligned}$$

From this expression of the decay amplitude, for the unpolarized differential decay width we get the following result:

$$\frac{d\Gamma}{d\hat{s}}(B \rightarrow \rho \ell^+ \ell^-) = \frac{G^2 \alpha^2 m_B}{2^{14} \pi^5} |V_{tb} V_{td}^*|^2 \lambda^{1/2}(1, \hat{r}, \hat{s}) v \Delta(\hat{s}) , \quad (14)$$

with

$$\begin{aligned} \Delta = & \frac{2m_B^2}{3\hat{r}_\rho \hat{s}} \text{Re} \left\{ -6m_B \hat{m}_\ell \hat{s} \lambda (B_1 - D_1)(B_4^* - B_5^*) \right. \\ & - 12m_B^2 \hat{m}_\ell^2 \hat{s} \lambda [B_4 B_5^* + (B_3 - D_2 - D_3)B_1^* - (B_2 + B_3 - D_3)D_1^*] \\ & + 6m_B^3 \hat{m}_\ell \hat{s} (1 - \hat{r}_\rho) \lambda (B_2 - D_2)(B_4^* - B_5^*) \\ & + 12m_B^4 \hat{m}_\ell^2 \hat{s} (1 - \hat{r}_\rho) \lambda (B_2 - D_2)(B_3^* - D_3^*) \\ & + 6m_B^3 \hat{m}_\ell \lambda \hat{s}^2 (B_4 - B_5)(B_3^* - D_3^*) \\ & + 48\hat{m}_\ell^2 \hat{r}_\rho \hat{s} (3B_1 D_1^* + 2m_B^4 \lambda A_1 C_1^*) \\ & + 48m_B^5 \hat{m}_\ell \hat{s} \lambda^2 (B_2 + D_2) B_7^* C_{TE}^* \\ & - 16m_B^4 \hat{r}_\rho \hat{s} (\hat{m}_\ell^2 - \hat{s}) \lambda (|A_1|^2 + |C_1|^2) \\ & - m_B^2 \hat{s} (2\hat{m}_\ell^2 - \hat{s}) \lambda (|B_4|^2 + |B_5|^2) \\ & - 48m_B^3 \hat{m}_\ell \hat{s} (1 - \hat{r}_\rho - \hat{s}) \lambda [(B_1 + D_1) B_7^* C_{TE}^* + 2(B_2 + D_2) B_6^* C_{TE}^*] \\ & - 6m_B^4 \hat{m}_\ell^2 \hat{s} \lambda [2(2 + 2\hat{r}_\rho - \hat{s}) B_2 D_2^* - \hat{s} |(B_3 - D_3)|^2] \\ & + 96m_B \hat{m}_\ell \hat{s} (\lambda + 12\hat{r}_\rho \hat{s}) (B_1 + D_1) B_6^* C_{TE}^* \\ & + 8m_B^2 \hat{s}^2 [v^2 |C_T|^2 + 4(3 - 2v^2) |C_{TE}|^2] [4(\lambda + 12\hat{r}_\rho \hat{s}) |B_6|^2 \\ & - 4m_B^2 \lambda (1 - \hat{r}_\rho - \hat{s}) B_6 B_7^* + m_B^4 \lambda^2 |B_7|^2] \\ & - 4m_B^2 \lambda [\hat{m}_\ell^2 (2 - 2\hat{r}_\rho + \hat{s}) + \hat{s} (1 - \hat{r}_\rho - \hat{s})] (B_1 B_2^* + D_1 D_2^*) \\ & + \hat{s} [6\hat{r}_\rho \hat{s} (3 + v^2) + \lambda (3 - v^2)] (|B_1|^2 + |D_1|^2) \\ & - 2m_B^4 \lambda \{ \hat{m}_\ell^2 [\lambda - 3(1 - \hat{r}_\rho)^2] - \lambda \hat{s} \} (|B_2|^2 + |D_2|^2) \\ & + 128m_B^2 \{ 4\hat{m}_\ell^2 [20\hat{r}_\rho \lambda - 12\hat{r}_\rho (1 - \hat{r}_\rho)^2 - \lambda \hat{s}] \\ & + \hat{s} [4\hat{r}_\rho \lambda + 12\hat{r}_\rho (1 - \hat{r}_\rho)^2 + \lambda \hat{s}] \} |C_T|^2 |t_1|^2 \\ & + 512m_B^2 \{ \hat{s} [4\hat{r}_\rho \lambda + 12\hat{r}_\rho (1 - \hat{r}_\rho)^2 + \lambda \hat{s}] \\ & + 8\hat{m}_\ell^2 [12\hat{r}_\rho (1 - \hat{r}_\rho)^2 + \lambda (\hat{s} - 8\hat{r}_\rho)] \} |C_{TE}|^2 |t_1|^2 \\ & - 64m_B^2 \hat{s}^2 [v^2 |C_T|^2 + 4(3 - 2v^2) |C_{TE}|^2] \{ 2[\lambda + 12\hat{r}_\rho (1 - \hat{r}_\rho)] B_6 t_1^* \\ & - m_B^2 \lambda (1 + 3\hat{r}_\rho - \hat{s}) B_7 t_1^* \} \\ & + 768m_B^3 \hat{m}_\ell \hat{r}_\rho \hat{s} \lambda (A_1 + C_1) C_T^* t_1^* \\ & - 192m_B \hat{m}_\ell \hat{s} [\lambda + 12\hat{r}_\rho (1 - \hat{r}_\rho)] (B_1 + D_1) C_{TE}^* t_1^* \\ & \left. + 192m_B^3 \hat{m}_\ell \hat{s} \lambda (1 + 3\hat{r}_\rho - \hat{s}) \lambda (B_2 + D_2) C_{TE}^* t_1^* \right\} , \quad (15) \end{aligned}$$

where $\hat{s} = q^2/m_B^2$, $\hat{r}_\rho = m_\rho^2/m_B^2$ and $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$, $\hat{m}_\ell = m_\ell/m_B$, $v = \sqrt{1 - 4\hat{m}_\ell^2/\hat{s}}$ is the final lepton velocity.

Using the matrix element for the $B \rightarrow \rho \ell^+ \ell^-$ decay, our next problem is to calculate the nine double-lepton polarization asymmetries. For this aim we introduce the spin projection operators

$$\Lambda_1 = \frac{1}{2}(1 + \gamma_5 \not{s}_i^-),$$

$$\Lambda_2 = \frac{1}{2}(1 + \gamma_5 \not{s}_i^+)$$

for the lepton ℓ^- and anti-lepton ℓ^+ , where $i = L, N, T$ correspond to the longitudinal, normal and transversal polarizations, respectively. Firstly we define the following orthogonal unit vectors $s_i^{\pm\mu}$ in the rest frame of ℓ^\pm (see also [1, 3, 22, 23]),

$$\begin{aligned} s_L^{-\mu} &= (0, \vec{e}_L^-) = \left(0, \frac{\vec{p}_-}{|\vec{p}_-|}\right), \\ s_N^{-\mu} &= (0, \vec{e}_N^-) = \left(0, \frac{\vec{p}_\rho \times \vec{p}_-}{|\vec{p}_\rho \times \vec{p}_-|}\right), \\ s_T^{-\mu} &= (0, \vec{e}_T^-) = (0, \vec{e}_N^- \times \vec{e}_L^-), \\ s_L^{+\mu} &= (0, \vec{e}_L^+) = \left(0, \frac{\vec{p}_+}{|\vec{p}_+|}\right), \\ s_N^{+\mu} &= (0, \vec{e}_N^+) = \left(0, \frac{\vec{p}_\rho \times \vec{p}_+}{|\vec{p}_\rho \times \vec{p}_+|}\right), \\ s_T^{+\mu} &= (0, \vec{e}_T^+) = (0, \vec{e}_N^+ \times \vec{e}_L^+), \end{aligned} \quad (16)$$

where \vec{p}_\mp and \vec{p}_ρ are the three-momenta of the leptons ℓ^\mp and ρ meson in the center of mass frame (CM) of $\ell^- \ell^+$ system, respectively. Transformation of unit vectors from the rest frame of the leptons to CM frame of leptons can be done by the Lorentz boost. Boosting of the longitudinal unit vectors $s_L^{\pm\mu}$ leads to

$$(s_L^{\mp\mu})_{CM} = \left(\frac{|\vec{p}_\mp|}{m_\ell}, \frac{E_\ell \vec{p}_\mp}{m_\ell |\vec{p}_\mp|}\right), \quad (17)$$

where $\vec{p}_+ = -\vec{p}_-$, E_ℓ and m_ℓ are the energy and mass of leptons in the CM frame, respectively. The remaining two unit vectors $s_N^{\pm\mu}$, $s_T^{\pm\mu}$ are unchanged under Lorentz boost.

We can now define the double-lepton polarization asymmetries as in [8]:

$$P_{ij}(\hat{s}) = \frac{\left(\frac{d\Gamma}{d\hat{s}}(\vec{s}_i^-, \vec{s}_j^+) - \frac{d\Gamma}{d\hat{s}}(-\vec{s}_i^-, \vec{s}_j^+)\right) - \left(\frac{d\Gamma}{d\hat{s}}(\vec{s}_i^-, -\vec{s}_j^+) - \frac{d\Gamma}{d\hat{s}}(-\vec{s}_i^-, -\vec{s}_j^+)\right)}{\left(\frac{d\Gamma}{d\hat{s}}(\vec{s}_i^-, \vec{s}_j^+) + \frac{d\Gamma}{d\hat{s}}(-\vec{s}_i^-, \vec{s}_j^+)\right) + \left(\frac{d\Gamma}{d\hat{s}}(\vec{s}_i^-, -\vec{s}_j^+) + \frac{d\Gamma}{d\hat{s}}(-\vec{s}_i^-, -\vec{s}_j^+)\right)}, \quad (18)$$

where $i, j = L, N, T$, and the first subindex i corresponds lepton while the second subindex j corresponds to antilepton, respectively.

After lengthy calculations we get the following results for the double-polarization asymmetries.

$$\begin{aligned}
P_{LL} = & \frac{m_B^2}{3\hat{r}_\rho\hat{s}\Delta} \text{Re} \left\{ -12m_B\hat{m}_\ell\hat{s}\lambda(B_1 - D_1)(B_4^* - B_5^*) \right. \\
& - 24m_B^2\hat{m}_\ell^2\hat{s}\lambda[B_4B_5^* + (B_1 - D_1)(B_3^* - D_3^*)] \\
& + 12m_B^3\hat{m}_\ell\hat{s}\lambda(1 - \hat{r}_\rho)[(B_2 - D_2)(B_4^* - B_5^*) + 2m_B\hat{m}_\ell(B_2 - D_2)(B_3^* - D_3^*)] \\
& - 32m_B^5\hat{m}_\ell\hat{s}\lambda^2(B_2 + D_2)B_7^*C_{TE}^* \\
& + 3m_B^2\hat{s}^2\lambda(1 + v^2)(|B_4|^2 + |B_5|^2) \\
& - 8m_B^4\hat{r}_\rho\hat{s}^2\lambda(1 + 3v^2)(|A_1|^2 + |C_1|^2) \\
& + 12m_B^3\hat{m}_\ell\hat{s}^2\lambda(B_3 - D_3)(B_4^* - B_5^*) \\
& + 12m_B^4\hat{m}_\ell^2\hat{s}^2\lambda|B_3 - D_3|^2 \\
& + 32m_B^3\hat{m}_\ell\hat{s}\lambda(1 - \hat{r}_\rho - \hat{s})[(B_1 + D_1)B_7^*C_{TE}^* + 2(B_2 + D_2)B_6^*C_{TE}^*] \\
& + 8m_B^2\hat{m}_\ell^2\lambda(4 - 4\hat{r}_\rho - \hat{s})(B_1D_2^* + B_2D_1^*) \\
& - 64m_B\hat{m}_\ell\hat{s}(\lambda + 12\hat{r}_\rho\hat{s})(B_1 + D_1)B_6^*C_{TE}^* \\
& - 16m_B^2\hat{s}[4\hat{m}_\ell^2(|C_T|^2 + 8|C_{TE}|^2) - \hat{s}(|C_T|^2 + 4|C_{TE}|^2)] [m_B^4\lambda^2|B_7|^2 + 4(\lambda + 12\hat{r}_\rho\hat{s})|B_6|^2] \\
& - 32\hat{m}_\ell^2(\lambda + 3\hat{r}_\rho\hat{s})B_1D_1^* \\
& - 8m_B^4\hat{m}_\ell^2\lambda[\lambda + 3(1 - \hat{r}_\rho)^2]B_2D_2^* \\
& + 8m_B^2\lambda[\hat{s} - \hat{s}(\hat{r}_\rho + \hat{s}) - 3\hat{m}_\ell^2(2 - 2\hat{r}_\rho - \hat{s})](B_1B_2^* + D_1D_2^*) \\
& - 64m_B^4\hat{s}^2\lambda(1 - \hat{r}_\rho - \hat{s})[v^2|C_T|^2 - 4(1 - 2v^2)|C_{TE}|^2]B_6B_7^* \\
& - m_B^4\hat{s}\lambda[\lambda(1 + 3v^2) - 3(1 - \hat{r}_\rho)^2(1 - v^2)](|B_2|^2 + |D_2|^2) \\
& + 4[6\hat{m}_\ell^2(\lambda + 6\hat{r}_\rho\hat{s}) - \hat{s}(\lambda + 12\hat{r}_\rho\hat{s})](|B_1|^2 + |D_1|^2) \\
& - 1024m_B^2\{12\hat{r}_\rho\hat{s}(1 - \hat{r}_\rho)^2(1 - 2v^2) - \lambda\hat{s}[4\hat{r}_\rho - \hat{s}(1 - 2v^2)]\}|t_1|^2|C_{TE}|^2 \\
& + 256m_B^2\hat{s}\{\lambda(\hat{s}v^2 - 8\hat{r}_\rho) + 12\hat{r}_\rho v^2[\lambda + (1 - \hat{r}_\rho)^2]\}|t_1|^2|C_T|^2 \\
& - 256m_B^2\hat{s}^2[\lambda + 12\hat{r}_\rho(1 - \hat{r}_\rho)][v^2|C_T|^2 - 4(1 - 2v^2)|C_{TE}|^2]B_6t_1^* \\
& + 128m_B^4\hat{s}^2\lambda(1 + 3\hat{r}_\rho - \hat{s})[v^2|C_T|^2 - 4(1 - 2v^2)|C_{TE}|^2]B_7t_1^* \\
& + 128m_B\hat{m}_\ell\hat{s}[\lambda + 12\hat{r}_\rho(1 - \hat{r}_\rho)](B_1 + D_1)t_1^*C_{TE}^* \\
& - 128m_B^3\hat{m}_\ell\hat{s}\lambda(1 + 3\hat{r}_\rho - \hat{s})(B_2 + D_2)t_1^*C_{TE}^* \\
& - 512m_B^3\hat{m}_\ell\hat{r}_\rho\hat{s}\lambda(A_1 + C_1)t_1^*C_T^* \\
& \left. - 64m_B^4\hat{m}_\ell^2\hat{r}_\rho\hat{s}\lambda A_1C_1^* \right\}, \tag{19}
\end{aligned}$$

$$\begin{aligned}
P_{LN} = & \frac{\pi m_B^2}{2\hat{r}_\rho\Delta} \sqrt{\frac{\lambda}{\hat{s}}} \text{Im} \left\{ 4m_B^2\hat{m}_\ell^2\lambda[B_2B_4^* + B_5D_2^* + 8(B_1 - D_1)B_7^*C_{TE}^*] \right. \\
& - 4m_B^4\hat{m}_\ell\lambda(1 - \hat{r}_\rho)[B_2D_2^* + 8m_B\hat{m}_\ell(B_2 - D_2)B_7^*C_{TE}^*] \\
& \left. + 2m_B^4\hat{m}_\ell\hat{s}\lambda[B_2B_3^* - 8(B_4 - B_5)B_7^*C_{TE}^* - 16m_B\hat{m}_\ell(B_3 - D_3)B_7^*C_{TE}^*] \right\}
\end{aligned}$$

$$\begin{aligned}
& - 2m_B^4 \hat{m}_\ell \hat{s} \lambda [B_3 D_2^* + (B_2 + D_2) D_3^*] \\
& - 2m_B^2 \hat{m}_\ell \hat{s} (1 + 3\hat{r}_\rho - \hat{s}) (B_1 B_2^* - D_1 D_2^* - 32B_5 C_{TE}^* t_1^*) \\
& + 32m_B^3 \hat{r}_\rho \hat{s}^2 v^2 [(A_1 - C_1) B_6^* C_T^* - 2(A_1 + C_1) B_6^* C_{TE}^*] \\
& - m_B^3 \hat{s} \lambda (1 + v^2) (B_2 B_5^* + B_4 D_2^*) \\
& - 4\hat{m}_\ell (1 - \hat{r}_\rho - \hat{s}) \{ B_1 D_1^* + m_B \hat{m}_\ell [B_1 (B_4^* + 16B_6^* C_{TE}^*) - D_1 (B_5^* + 16B_6^* C_{TE}^*)] \} \\
& + 64m_B^3 \hat{m}_\ell^2 (1 - \hat{r}_\rho) (1 - \hat{r}_\rho - \hat{s}) (B_2 - D_2) B_6^* C_{TE}^* \\
& - 2m_B^2 \hat{m}_\ell \hat{s} (1 - \hat{r}_\rho - \hat{s}) (B_1 + D_1) (B_3^* - D_3^*) \\
& + 32m_B^2 \hat{m}_\ell \hat{s} (1 - \hat{r}_\rho - \hat{s}) [(B_4 - B_5) B_6^* C_{TE}^* + 2m_B \hat{m}_\ell (B_3 - D_3) B_6^* C_{TE}^*] \\
& + m_B \hat{s} (1 - \hat{r}_\rho - \hat{s}) (1 + v^2) (B_1 B_5^* + B_4 D_1^*) \\
& + 2m_B^2 \hat{m}_\ell [\lambda + (1 - \hat{r}_\rho) (1 - \hat{r}_\rho - \hat{s})] (B_2 D_1^* + B_1 D_2^*) \\
& - 128m_B^3 \hat{m}_\ell^2 (1 - \hat{r}_\rho) (1 + 3\hat{r}_\rho - \hat{s}) (B_2 - D_2) t_1^* C_{TE}^* \\
& - 64m_B^2 \hat{m}_\ell \hat{s} (1 + 3\hat{r}_\rho - \hat{s}) [B_4 C_{TE}^* t_1^* + 2m_B \hat{m}_\ell (B_3 - D_3) C_{TE}^* t_1^*] \\
& - 64m_B^3 \hat{r}_\rho \hat{s} (1 - \hat{r}_\rho) v^2 [A_1 (C_T^* - 2C_{TE}^*) t_1^* - C_1 (C_T^* + 2C_{TE}^*) t_1^*] \\
& - 32m_B \hat{s} [(1 + 3\hat{r}_\rho - \hat{s}) D_1 C_{TE}^* t_1^* + 2\hat{r}_\rho v^2 D_1 C_T^* t_1^* - (1 - \hat{r}_\rho - \hat{s}) v^2 D_1 C_{TE}^* t_1^*] \\
& + 32m_B \hat{s} [(1 + 3\hat{r}_\rho - \hat{s}) B_1 C_{TE}^* t_1^* - 2\hat{r}_\rho v^2 B_1 C_T^* t_1^* - (1 - \hat{r}_\rho - \hat{s}) v^2 B_1 C_{TE}^* t_1^*] \} , \tag{20}
\end{aligned}$$

$$\begin{aligned}
P_{NL} = & \frac{\pi m_B^2}{2\hat{r}_\rho \Delta} \sqrt{\frac{\lambda}{\hat{s}}} \text{Im} \left\{ 4m_B^3 \hat{m}_\ell^2 \lambda [B_2 B_5^* + B_4 D_2^* - 8(B_1 - D_1) B_7^* C_{TE}^*] \right. \\
& + 4m_B^4 \hat{m}_\ell \lambda (1 - \hat{r}_\rho) [B_2 D_2^* + 8m_B \hat{m}_\ell (B_2 - D_2) B_7^* C_{TE}^*] \\
& - 2m_B^4 \hat{m}_\ell \hat{s} \lambda [B_2 B_3^* - 8(B_4 - B_5) B_7^* C_{TE}^* - 16m_B \hat{m}_\ell (B_3 - D_3) B_7^* C_{TE}^*] \\
& + 2m_B^4 \hat{m}_\ell \hat{s} \lambda [B_3 D_2^* + (B_2 + D_2) D_3^*] \\
& + 2m_B^2 \hat{m}_\ell \hat{s} (1 + 3\hat{r}_\rho - \hat{s}) (B_1 B_2^* - D_1 D_2^* - 32B_5 C_{TE}^* t_1^*) \\
& - 32m_B^3 \hat{r}_\rho \hat{s}^2 v^2 [(A_1 - C_1) B_6^* C_T^* + 2(A_1 + C_1) B_6^* C_{TE}^*] \\
& - m_B^3 \hat{s} \lambda (1 + v^2) (B_2 B_4^* + B_5 D_2^*) \\
& + 4\hat{m}_\ell (1 - \hat{r}_\rho - \hat{s}) \{ B_1 D_1^* - m_B \hat{m}_\ell [B_1 (B_5^* - 16B_6^* C_{TE}^*) - D_1 (B_4^* - 16B_6^* C_{TE}^*)] \} \\
& + 64m_B^3 \hat{m}_\ell^2 (1 - \hat{r}_\rho) (1 - \hat{r}_\rho - \hat{s}) (B_2 - D_2) B_6^* C_{TE}^* \\
& + 2m_B^2 \hat{m}_\ell \hat{s} (1 - \hat{r}_\rho - \hat{s}) (B_1 + D_1) (B_3^* - D_3^*) \\
& - 32m_B^2 \hat{m}_\ell \hat{s} (1 - \hat{r}_\rho - \hat{s}) [(B_4 - B_5) B_6^* C_{TE}^* + 2m_B \hat{m}_\ell (B_3 - D_3) B_6^* C_{TE}^*] \\
& + m_B \hat{s} (1 - \hat{r}_\rho - \hat{s}) (1 + v^2) (B_1 B_4^* + B_5 D_1^*) \\
& - 2m_B^2 \hat{m}_\ell [\lambda + (1 - \hat{r}_\rho) (1 - \hat{r}_\rho - \hat{s})] (B_2 D_1^* + B_1 D_2^*) \\
& + 128m_B^3 \hat{m}_\ell^2 (1 - \hat{r}_\rho) (1 + 3\hat{r}_\rho - \hat{s}) (B_2 - D_2) t_1^* C_{TE}^* \\
& + 64m_B^2 \hat{m}_\ell \hat{s} (1 + 3\hat{r}_\rho - \hat{s}) [B_4 C_{TE}^* t_1^* + 2m_B \hat{m}_\ell (B_3 - D_3) C_{TE}^* t_1^*] \}
\end{aligned}$$

$$\begin{aligned}
& + 64m_B^3\hat{r}_\rho\hat{s}(1-\hat{r}_\rho)v^2\left[A_1(C_T^*+2C_{TE}^*)t_1^*-C_1(C_T^*-2C_{TE}^*)t_1^*\right] \\
& - 32m_B\hat{s}\left[(1+3\hat{r}_\rho-\hat{s})B_1C_{TE}^*t_1^*+2\hat{r}_\rho v^2B_1C_T^*t_1^*-(1-\hat{r}_\rho-\hat{s})v^2B_1C_{TE}^*t_1^*\right] \\
& + 32m_B\hat{s}\left[(1+3\hat{r}_\rho-\hat{s})D_1C_{TE}^*t_1^*-2\hat{r}_\rho v^2D_1C_T^*t_1^*-(1-\hat{r}_\rho-\hat{s})v^2D_1C_{TE}^*t_1^*\right]\Big\}, \tag{21}
\end{aligned}$$

$$\begin{aligned}
P_{LT} = & \frac{\pi m_B^2 v}{\hat{r}_\rho \Delta} \sqrt{\frac{\lambda}{\hat{s}}} \text{Re}\left\{m_B^4 \hat{m}_\ell \lambda (1-\hat{r}_\rho) |B_2 - D_2|^2 \right. \\
& - 8m_B^2 \hat{m}_\ell \hat{r}_\rho \hat{s} (A_1 B_1^* - C_1 D_1^*) \\
& - m_B^3 \hat{s} \lambda (B_2 B_5^* + B_4 D_2^* - m_B \hat{m}_\ell B_2 B_3^*) \\
& - 8m_B^4 \hat{m}_\ell \hat{s} \lambda (B_4 + B_5) B_7^* C_{TE}^* \\
& - m_B^4 \hat{m}_\ell \hat{s} \lambda (B_2 D_3^* + B_3 D_2^* - D_2 D_3^*) \\
& + 16m_B^3 \hat{r}_\rho \hat{s}^2 [A_1 B_6^* (C_T^* - 2C_{TE}^*) + C_1 B_6^* (C_T^* + 2C_{TE}^*)] \\
& + \hat{m}_\ell (1-\hat{r}_\rho-\hat{s}) |B_1 - D_1|^2 \\
& + m_B \hat{s} (1-\hat{r}_\rho-\hat{s}) [B_1 B_5^* + B_4 D_1^* + 16m_B \hat{m}_\ell (B_4 + B_5) B_6^* C_{TE}^* \\
& - m_B \hat{m}_\ell (B_1 - D_1) (B_3^* - D_3^*)] \\
& - m_B^2 \hat{m}_\ell [\lambda + (1-\hat{r}_\rho)(1-\hat{r}_\rho-\hat{s})] (B_1 - D_1) (B_2^* - D_2^*) \\
& - 1024m_B^2 \hat{m}_\ell \hat{r}_\rho (1-\hat{r}_\rho) (|C_T|^2 + 4|C_{TE}|^2) |t_1|^2 \\
& + 512m_B^2 \hat{m}_\ell \hat{r}_\rho \hat{s} (|C_T|^2 + 4|C_{TE}|^2) B_6 t_1^* \\
& - 32m_B^2 \hat{m}_\ell \hat{s} (1+3\hat{r}_\rho-\hat{s}) (B_4 + B_5) C_{TE}^* t_1^* \\
& - 32m_B^3 \hat{r}_\rho \hat{s} (1-\hat{r}_\rho) [A_1 (C_T^* - 2C_{TE}^*) t_1^* + C_1 (C_T^* + 2C_{TE}^*) t_1^*] \\
& \left. - 32m_B \hat{r}_\rho \hat{s} [B_1 (C_T^* - 2C_{TE}^*) t_1^* - D_1 (C_T^* + 2C_{TE}^*) t_1^*] \right\}, \tag{22}
\end{aligned}$$

$$\begin{aligned}
P_{TL} = & \frac{\pi m_B^2 v}{\hat{r}_\rho \Delta} \sqrt{\frac{\lambda}{\hat{s}}} \text{Re}\left\{m_B^4 \hat{m}_\ell \lambda (1-\hat{r}_\rho) |B_2 - D_2|^2 \right. \\
& + 8m_B^2 \hat{m}_\ell \hat{r}_\rho \hat{s} (A_1 B_1^* - C_1 D_1^*) \\
& + m_B^3 \hat{s} \lambda (B_2 B_4^* + B_5 D_2^* + m_B \hat{m}_\ell B_2 B_3^*) \\
& + 8m_B^4 \hat{m}_\ell \hat{s} \lambda (B_4 + B_5) B_7^* C_{TE}^* \\
& - m_B^4 \hat{m}_\ell \hat{s} \lambda (B_2 D_3^* + B_3 D_2^* - D_2 D_3^*) \\
& + 16m_B^3 \hat{r}_\rho \hat{s}^2 [A_1 B_6^* (C_T^* + 2C_{TE}^*) + C_1 B_6^* (C_T^* - 2C_{TE}^*)] \\
& + \hat{m}_\ell (1-\hat{r}_\rho-\hat{s}) |B_1 - D_1|^2 \\
& - m_B \hat{s} (1-\hat{r}_\rho-\hat{s}) [B_1 B_4^* + B_5 D_1^* + 16m_B \hat{m}_\ell (B_4 + B_5) B_6^* C_{TE}^* \\
& + m_B \hat{m}_\ell (B_1 - D_1) (B_3^* - D_3^*)] \\
& \left. - m_B^2 \hat{m}_\ell [\lambda + (1-\hat{r}_\rho)(1-\hat{r}_\rho-\hat{s})] (B_1 - D_1) (B_2^* - D_2^*) \right\}
\end{aligned}$$

$$\begin{aligned}
& - 1024m_B^2\hat{m}_\ell\hat{r}_\rho(1-\hat{r}_\rho)\left(|C_T|^2 + 4|C_{TE}|^2\right)|t_1|^2 \\
& + 512m_B^2\hat{m}_\ell\hat{r}_\rho\hat{s}\left(|C_T|^2 + 4|C_{TE}|^2\right)B_6t_1^* \\
& + 32m_B^2\hat{m}_\ell\hat{s}(1+3\hat{r}_\rho-\hat{s})(B_4+B_5)C_{TE}^*t_1^* \\
& - 32m_B^3\hat{r}_\rho\hat{s}(1-\hat{r}_\rho)\left[A_1(C_T^*+2C_{TE}^*)t_1^*+C_1(C_T^*-2C_{TE}^*)t_1^*\right] \\
& + 32m_B\hat{r}_\rho\hat{s}\left[B_1(C_T^*+2C_{TE}^*)t_1^*-D_1(C_T^*-2C_{TE}^*)t_1^*\right]\Big\}, \tag{23}
\end{aligned}$$

$$\begin{aligned}
P_{NT} = \frac{2m_B^2v}{3\hat{r}_\rho\Delta} \text{Im}\Big\{ & 4\lambda\left\{B_1D_1^*+m_B^4\lambda\left[B_2D_2^*-2m_B\hat{m}_\ell B_2B_7^*(C_T^*-4C_{TE}^*)\right.\right. \\
& - 2m_B\hat{m}_\ell D_2B_7^*(C_T^*+4C_{TE}^*)\Big]\Big\} \\
& - 6m_B\hat{m}_\ell\lambda(B_1-D_1)(B_4^*+B_5^*) \\
& + 6m_B^3\hat{m}_\ell\lambda(1-\hat{r}_\rho)(B_2-D_2)(B_4^*+B_5^*) \\
& + 6m_B^3\hat{m}_\ell\hat{s}\lambda(B_3-D_3)(B_4^*+B_5^*) \\
& - 4m_B^2\lambda(1-\hat{r}_\rho-\hat{s})\left[B_1D_2^*+B_2D_1^*+32m_B^2\hat{s}\text{Re}[B_6B_7^*]C_TC_{TE}^*\right] \\
& + 8m_B^3\hat{m}_\ell\lambda(1-\hat{r}_\rho-\hat{s})\left[(B_1B_7^*+2B_2B_6^*)(C_T^*-4C_{TE}^*)\right. \\
& + (B_7^*D_1+2B_6^*D_2)(C_T^*+4C_{TE}^*)\Big] \\
& + 32m_B^2\hat{s}\left[4(\lambda+12\hat{r}_\rho\hat{s})|B_6|^2+\lambda^2m_B^4|B_7|^2\right]C_TC_{TE}^* \\
& + 2m_B^2\hat{s}\lambda\left(3B_4B_5^*-8m_B^2\hat{r}_\rho A_1C_1^*\right) \\
& - 16m_B\hat{m}_\ell\left\{\lambda\left[B_1B_6^*(C_T^*-4C_{TE}^*)+D_1B_6^*(C_T^*+4C_{TE}^*)\right]+12\hat{r}_\rho\hat{s}(B_1+D_1)B_6^*C_T^*\right\} \\
& + 32m_B\hat{m}_\ell\left\{12\hat{r}_\rho(1-\hat{r}_\rho)(B_1+D_1)C_T^*t_1^*+\lambda\left[B_1(C_T^*-4C_{TE}^*)t_1^*+D_1(C_T^*+4C_{TE}^*)t_1^*\right]\right\} \\
& - 256m_B^3\hat{m}_\ell\hat{r}_\rho\lambda\left[A_1(C_T^*+2C_{TE}^*)t_1^*-C_1(C_T^*-2C_{TE}^*)t_1^*\right] \\
& - 32m_B^3\hat{m}_\ell\lambda(1+3\hat{r}_\rho-\hat{s})\left[B_2(C_T^*-4C_{TE}^*)t_1^*+D_2(C_T^*+4C_{TE}^*)t_1^*\right] \\
& + 256m_B^2\hat{s}\left\{2[\lambda+12\hat{r}_\rho(2+2\hat{r}_\rho-\hat{s})]|t_1|^2-2[\lambda+12\hat{r}_\rho(1-\hat{r}_\rho)]\text{Re}[B_6t_1^*]\right. \\
& + m_B^2\lambda(1+3\hat{r}_\rho-\hat{s})\text{Re}[B_7t_1^*]\Big\}C_TC_{TE}^*\Big\}, \tag{24}
\end{aligned}$$

$$\begin{aligned}
P_{TN} = \frac{2m_B^2v}{3\hat{r}_\rho\Delta} \text{Im}\Big\{ & - 4\lambda\left\{B_1D_1^*+m_B^4\lambda\left[B_2D_2^*+2m_B\hat{m}_\ell B_2B_7^*(C_T^*+4C_{TE}^*)\right.\right. \\
& + 2m_B\hat{m}_\ell D_2B_7^*(C_T^*-4C_{TE}^*)\Big]\Big\} \\
& - 6m_B\hat{m}_\ell\lambda(B_1-D_1)(B_4^*+B_5^*) \\
& + 6m_B^3\hat{m}_\ell\lambda(1-\hat{r}_\rho)(B_2-D_2)(B_4^*+B_5^*) \\
& + 6m_B^3\hat{m}_\ell\hat{s}\lambda(B_3-D_3)(B_4^*+B_5^*) \\
& + 4m_B^2\lambda(1-\hat{r}_\rho-\hat{s})\left[B_1D_2^*+B_2D_1^*-32m_B^2\hat{s}\text{Re}[B_6B_7^*]C_TC_{TE}^*\right] \\
& + 8m_B^3\hat{m}_\ell\lambda(1-\hat{r}_\rho-\hat{s})\left[(B_1B_7^*+2B_2B_6^*)(C_T^*+4C_{TE}^*)\right. \\
& + (B_7^*D_1+2B_6^*D_2)(C_T^*-4C_{TE}^*)\Big]
\end{aligned}$$

$$\begin{aligned}
& + 32m_B^2\hat{s}\left[4(\lambda + 12\hat{r}_\rho\hat{s})|B_6|^2 + \lambda^2m_B^4|B_7|^2\right]C_TC_{TE}^* \\
& + 2m_B^2\hat{s}\lambda\left(3B_4B_5^* + 8m_B^2\hat{r}_\rho A_1C_1^*\right) \\
& - 16m_B\hat{m}_\ell\left\{\lambda\left[B_1B_6^*(C_T^* + 4C_{TE}^*) + D_1B_6^*(C_T^* - 4C_{TE}^*)\right] + 12\hat{r}_\rho\hat{s}(B_1 + D_1)B_6^*C_T^*\right\} \\
& + 32m_B\hat{m}_\ell\left\{12\hat{r}_\rho(1 - \hat{r}_\rho)(B_1 + D_1)C_T^*t_1^* + \lambda\left[B_1(C_T^* + 4C_{TE}^*)t_1^* + D_1(C_T^* - 4C_{TE}^*)t_1^*\right]\right\} \\
& + 256m_B^3\hat{m}_\ell\hat{r}_\rho\lambda\left[A_1(C_T^* - 2C_{TE}^*)t_1^* - C_1(C_T^* + 2C_{TE}^*)t_1^*\right] \\
& - 32m_B^3\hat{m}_\ell\lambda(1 + 3\hat{r}_\rho - \hat{s})\left[B_2(C_T^* + 4C_{TE}^*)t_1^* + D_2(C_T^* - 4C_{TE}^*)t_1^*\right] \\
& + 256m_B^2\hat{s}\left\{2[\lambda + 12\hat{r}_\rho(2 + 2\hat{r}_\rho - \hat{s})]|t_1|^2 - 2[\lambda + 12\hat{r}_\rho(1 - \hat{r}_\rho)]\text{Re}[B_6t_1^*]\right. \\
& \left. + m_B^2\lambda(1 + 3\hat{r}_\rho - \hat{s})\text{Re}[B_7t_1^*]\right\}C_TC_{TE}^* \Big\} , \tag{25}
\end{aligned}$$

$$\begin{aligned}
P_{NN} = & \frac{2m_B^2}{3\hat{r}_\rho\Delta}\text{Re}\Bigg\{ - 24\hat{m}_\ell^2\hat{r}_\rho(|B_1|^2 + |D_1|^2) \\
& - 6m_B\hat{m}_\ell\lambda(B_1 - D_1)(B_4^* - B_5^*) \\
& - 48m_B^5\hat{m}_\ell\lambda^2(B_2 + D_2)B_7^*C_{TE}^* \\
& + 6m_B^2\hat{m}_\ell^2\lambda\left[|B_4|^2 + |B_5|^2 - 2B_1(B_2^* + B_3^* - D_3^*) + 2D_1(B_3^* - D_2^* - D_3^*)\right] \\
& + 6m_B^3\hat{m}_\ell\lambda(1 - \hat{r}_\rho)\left[(B_2 - D_2)(B_4^* - B_5^*) + 2m_B\hat{m}_\ell(B_2 - D_2)(B_3^* - D_3^*)\right] \\
& + m_B^2\hat{s}\lambda\left[16m_B^2\hat{r}_\rho v^2 A_1C_1^* - 3(1 + v^2)B_4B_5^*\right] \\
& + 6m_B^4\hat{m}_\ell^2\lambda(2 + 2\hat{r}_\rho - \hat{s})(|B_2|^2 + |D_2|^2) \\
& + 6m_B^3\hat{m}_\ell\hat{s}\lambda(B_3 - D_3)(B_4^* - B_5^*) \\
& + 6m_B^4\hat{m}_\ell^2\hat{s}\lambda|B_3 - D_3|^2 \\
& + 48m_B^3\hat{m}_\ell\lambda(1 - \hat{r}_\rho - \hat{s})\left[(B_1 + D_1)B_7^*C_{TE}^* + 2(B_2 + D_2)B_6^*C_{TE}^*\right] \\
& - 96m_B\hat{m}_\ell(1 - \hat{r}_\rho - \hat{s})^2(B_1 + D_1)B_6^*C_{TE}^* \\
& + 8m_B^4\hat{s}\lambda\left[\lambda m_B^2|B_7|^2 - 4(1 - \hat{r}_\rho - \hat{s})B_6B_7^*\right]\left[v^2|C_T|^2 - 4(3 - 2v^2)|C_{TE}|^2\right] \\
& + m_B^2\lambda\left[3(2 - 2\hat{r}_\rho - \hat{s}) - v^2(2 - 2\hat{r}_\rho + \hat{s})\right](B_1D_2^* + B_2D_1^*) \\
& - m_B^4\lambda\left[(3 + v^2)\lambda + 3(1 - v^2)(1 - \hat{r}_\rho)^2\right]B_2D_2^* \\
& - 2[6\hat{r}_\rho\hat{s}(1 - v^2) + \lambda(3 - v^2)]B_1D_1^* \\
& + 32m_B^2\hat{s}\left\{(\lambda + 12\hat{r}_\rho\hat{s})v^2|C_T|^2 - 4[\lambda(3 - 2v^2) + 12\hat{r}_\rho\hat{s}]|C_{TE}|^2\right\}|B_6|^2 \\
& - 192m_B^3\hat{m}_\ell\lambda(1 + 3\hat{r}_\rho - \hat{s})(B_2 + D_2)C_{TE}^*t_1^* \\
& + 192m_B\hat{m}_\ell[\lambda + 4\hat{r}_\rho(1 - \hat{r}_\rho)](B_1 + D_1)C_{TE}^*t_1^* \\
& + 128m_B^2\hat{s}v^2[\lambda + 12\hat{r}_\rho(2 + 2\hat{r}_\rho - \hat{s})]|C_T|^2|t_1|^2 \\
& - 512m_B^2\hat{s}[\lambda(3 - 2v^2) + 12\hat{r}_\rho(2 + 2\hat{r}_\rho - \hat{s})]|C_{TE}|^2|t_1|^2 \\
& - 128m_B^2\hat{s}\left\{[\lambda + 12\hat{r}_\rho(1 - \hat{r}_\rho)]v^2|C_T|^2 - 4[\lambda(3 - 2v^2) + 12\hat{r}_\rho(1 - \hat{r}_\rho)]|C_{TE}|^2\right\}B_6t_1^* \\
& + 64m_B^4(1 + 3\hat{r}_\rho - \hat{s})\hat{s}\left[v^2|C_T|^2 - 4(3 - 2v^2)|C_{TE}|^2\right]B_7t_1^* \Big\} , \tag{26}
\end{aligned}$$

$$\begin{aligned}
P_{TT} = & \frac{2m_B^2}{3\hat{r}_\rho\hat{s}\Delta} \text{Re} \left\{ 8m_B^4\hat{r}_\rho\hat{s}\lambda \left[4\hat{m}_\ell^2(|A_1|^2 + |C_1|^2) + 2\hat{s}A_1C_1^* \right] \right. \\
& + 6m_B\hat{m}_\ell\hat{s}\lambda(B_1 - D_1)(B_4^* - B_5^*) \\
& - 16m_B^5\hat{m}_\ell\hat{s}\lambda^2(B_2 + D_2)B_7^*C_{TE}^* \\
& - 6m_B^2\hat{m}_\ell^2\hat{s}\lambda \left[|B_4|^2 + |B_5|^2 - 2(B_1 - D_1)(B_3^* - D_3^*) \right] \\
& - 6m_B^3\hat{m}_\ell\hat{s}\lambda(1 - \hat{r}_\rho) \left[(B_2 - D_2)(B_4^* - B_5^*) + 2m_B\hat{m}_\ell(B_2 - D_2)(B_3^* - D_3^*) \right] \\
& - 6m_B^3\hat{m}_\ell\hat{s}^2\lambda(B_3 - D_3)(B_4^* - B_5^*) \\
& - 6m_B^4\hat{m}_\ell^2\hat{s}^2\lambda|B_3 - D_3|^2 \\
& + 16m_B^3\hat{m}_\ell\hat{s}\lambda(1 - \hat{r}_\rho - \hat{s}) \left[(B_1 + D_1)B_7^*C_{TE}^* + 2(B_2 + D_2)B_6^*C_{TE}^* \right] \\
& + 4m_B^2\hat{m}_\ell^2\lambda(4 - 4\hat{r}_\rho - \hat{s})(B_1B_2^* + D_1D_2^*) \\
& + 2\hat{s}[6\hat{r}_\rho\hat{s}(1 - v^2) + \lambda(1 - 3v^2)]B_1D_1^* \\
& - 2m_B^4\hat{m}_\ell^2\lambda[\lambda + 3(1 - \hat{r}_\rho)^2](|B_2|^2 + |D_2|^2) \\
& - m_B^2\hat{s}\lambda[2 - 2\hat{r}_\rho + \hat{s} - 3v^2(2 - 2\hat{r}_\rho - \hat{s})](B_1D_2^* + B_2D_1^*) \\
& - 8\hat{m}_\ell^2(\lambda - 3\hat{r}_\rho\hat{s})(|B_1|^2 + |D_1|^2) \\
& - 32m_B\hat{m}_\ell\hat{s}(\lambda - 12\hat{r}_\rho\hat{s})(B_1 + D_1)B_6^*C_{TE}^* \\
& - 8m_B^4\hat{s}^2\lambda \left[\lambda m_B^2|B_7|^2 - 4(1 - \hat{r}_\rho - \hat{s})B_6B_7^* \right] \left[v^2|C_T|^2 + 4(1 - 2v^2)|C_{TE}|^2 \right] \\
& + 3m_B^2\hat{s}^2\lambda(1 + v^2)B_4B_5^* \\
& - m_B^4\hat{s}\lambda \left[(1 + 3v^2)\lambda - 3(1 - v^2)(1 - \hat{r}_\rho)^2 \right] B_2D_2^* \\
& - 32m_B^2\hat{s}^2 \left\{ (\lambda + 12\hat{r}_\rho\hat{s})v^2|C_T|^2 + 4[\lambda(1 - 2v^2) - 12\hat{r}_\rho\hat{s}]|C_{TE}|^2 \right\} |B_6|^2 \\
& - 128m_B^2 \left\{ 4\lambda[\lambda - (1 - \hat{r}_\rho)^2] + 8\hat{s}(1 - \hat{r}_\rho)[\lambda - 6\hat{r}_\rho(1 - \hat{r}_\rho)] + 8\lambda\hat{s}v^2(8\hat{r}_\rho - \hat{s}) \right\} |C_{TE}|^2 |t_1|^2 \\
& + 128m_B^2 \left\{ 16\lambda\hat{r}_\rho\hat{s} - \lambda[\lambda - (1 - \hat{r}_\rho)^2]v^2 - 2\hat{s}v^2[\lambda(1 + 3\hat{r}_\rho) + 6\hat{r}_\rho(1 - \hat{r}_\rho)^2] \right\} |C_T|^2 |t_1|^2 \\
& + 128m_B^2\hat{s}^2 \left\{ [\lambda + 12\hat{r}_\rho(1 - \hat{r}_\rho)]v^2|C_T|^2 + 4[\lambda(1 - 2v^2) - 12\hat{r}_\rho(1 - \hat{r}_\rho)] \right\} B_6t_1^* \\
& - 64m_B^4\hat{s}^2\lambda(1 + 3\hat{r}_\rho - \hat{s}) \left[v^2|C_T|^2 + 4(1 - 2v^2)|C_{TE}|^2 \right] B_7t_1^* \\
& + 64m_B\hat{m}_\ell\hat{s}[\lambda - 12\hat{r}_\rho(1 - \hat{r}_\rho)](B_1 + D_1)C_{TE}^*t_1^* \\
& + 512m_B^3\hat{m}_\ell\hat{r}_\rho\hat{s}\lambda(A_1 + C_1)C_T^*t_1^* \\
& \left. - 64m_B^3\hat{m}_\ell\hat{s}\lambda(1 + 3\hat{r}_\rho - \hat{s})(B_2 + D_2)C_{TE}^*t_1^* \right\}, \tag{27}
\end{aligned}$$

3 Numerical analysis

In this section we analyze the effects of the Wilson coefficients on the polarized FB asymmetry. The input parameters we use in our numerical calculations are: $m_\rho = 0.77 \text{ GeV}$, $m_\tau = 1.77 \text{ GeV}$, $m_\mu = 0.106 \text{ GeV}$, $m_b = 4.8 \text{ GeV}$, $m_B = 5.26 \text{ GeV}$ and $\Gamma_B = 4.22 \times 10^{-13} \text{ GeV}$. For the values of the Wilson coefficients we use $C_7^{SM} = -0.313$, $C_9^{SM} = 4.344$ and $C_{10}^{SM} = -4.669$. It should be noted that the above-presented value for C_9^{SM} corresponds only to short distance contributions. In addition to the short distance contributions, it receives long distance contributions which result from the conversion of $\bar{u}u$, $\bar{d}d$ and $\bar{c}c$ to

the lepton pair. In order to minimize the hadronic uncertainties we will discard the regions around low lying resonances ρ , w , J/ψ , ψ' , ψ'' , by dividing the q^2 region to low and high dilepton mass intervals:

$$\begin{aligned} \text{Region I: } & 1 \text{ GeV}^2 \leq q^2 \leq 8 \text{ GeV}^2 , \\ \text{Region II: } & 14.5 \text{ GeV}^2 \leq q^2 \leq (m_B - m_\rho)^2 \text{ GeV}^2 , \end{aligned}$$

where the contributions of the higher ψ resonances do still exist in the second region. The form factors we have used in the present work are more refined ones predicted by the light cone QCD sum rules [24]. The q^2 dependence of the form factors for the $B \rightarrow \rho$ transition can be represented in the following form:

$$F(q^2) = \frac{r_1}{1 - q^2/m_{\text{res}}^2} + \frac{r_2}{1 - q^2/m_{\text{fit}}^2} , \quad (28)$$

$$F(q^2) = \frac{r_2}{1 - q^2/m_{\text{fit}}^2} , \quad (29)$$

$$F(q^2) = \frac{r_1}{1 - q^2/m_{\text{fit}}^2} + \frac{r_2}{(1 - q^2/m_{\text{fit}}^2)^2} , \quad (30)$$

with the three independent parameters r_1 , r_2 and m_{fit} being listed listed in Table 1. The dominant poles at $q^2 = m_{\text{res}}^2$ correspond to the resonances

$$J^P = \begin{cases} 1^- & \text{for } V , \\ 0^- & \text{for } A_0 , \\ 1^+ & \text{for } A_1, A_2, A_3 \text{ and } T_2, T_3 . \end{cases}$$

The values of the parameters r_1 , r_2 and m_{fit} for various form factors are presented in Table-1.

	r_1	$m_{\text{res}}^2 \text{ (GeV}^2\text{)}$	r_2	$m_{\text{fit}}^2 \text{ (GeV}^2\text{)}$	Fit Eq.
$V^{B_q \rightarrow \rho}$	1.045	5.32 ²	-0.721	38.34	(28)
$A_0^{B_q \rightarrow \rho}$	1.527	5.28 ²	-1.220	33.36	(28)
$A_1^{B_q \rightarrow \rho}$	—	—	0.240	37.51	(29)
$A_2^{B_q \rightarrow \rho}$	0.009	—	0.212	40.82	(30)
$T_1^{B_q \rightarrow \rho}$	0.897	5.32 ²	-0.629	38.04	(28)
$T_2^{B_q \rightarrow \rho}$	—	—	0.267	38.59	(29)
$\tilde{T}_3^{B_q \rightarrow \rho}$	0.022	—	0.246	40.88	(30)

Table 1: $B \rightarrow \rho$ decay form factors in a three-parameter r_1 , r_2 and m_{fit} fit.

Note that T_3 entering into Eqs. (9) and (10) is related to \tilde{T}_3 as follows::

$$T_3 = \frac{m_B^2 - m_\rho^2}{q^2} (\tilde{T}_3 - T_2) .$$

In the numerical analysis the values of the new Wilson coefficients which describe the new physics beyond the SM are needed. In our calculations the new Wilson coefficients are

varied in the range $-|C_{10}^{SM}| \leq |C_X| \leq |C_{10}^{SM}|$. The experimental results on the branching ratio of the $B \rightarrow K^*(K)\ell^+\ell^-$ decay [25, 26] and the upper limit on the branching ratio of $B \rightarrow \mu^+\mu^-$ [27] suggests that this is the right order of magnitude for the new Wilson coefficients.

It follows from the expressions of all nine double-lepton polarization asymmetries that depend both on q^2 and the new Wilson coefficients C_X . Therefore, it may experimentally be difficult to study these dependencies at the same time. For this reason, we eliminate q^2 dependence by performing integration over q^2 in the allowed region, i.e., we consider the averaged double-lepton polarization asymmetries. The averaging over q^2 is defined as

$$\langle P_{ij} \rangle = \frac{\int_{R_i} P_{ij} \frac{d\mathcal{B}}{d\hat{s}} d\hat{s}}{\int_{R_i} \frac{d\mathcal{B}}{d\hat{s}} d\hat{s}},$$

where R_i = Regions I or II, over which the integrations are calculated. We present our analysis in a series of figures.

In Figs. (1) and (2) we present the dependence of $\langle P_{LL} \rangle$ on C_X for the $B \rightarrow \rho\mu^+\mu^-$ decay in the regions I and II, respectively. The intersection of all curves corresponds to the SM case. From these figures we see that $\langle P_{LL} \rangle$ exhibits strong dependence only on the tensor interactions C_T and C_{TE} , and has practically symmetric behavior in regard to its dependence on C_T and C_{TE} with respect to zero position. Furthermore, $\langle P_{LL} \rangle$ seems to be independent of all remaining new Wilson coefficients.

We depict from Figs. (3) and (4) the dependence of $\langle P_{LT} \rangle$ on C_X for the $B \rightarrow \rho\mu^+\mu^-$ decay in the regions I and II, respectively. We observe from these figures that $\langle P_{LT} \rangle$ is sensitive to the existence of scalar C_{LRRL} , C_{RRLL} and tensor interactions C_T , C_{TE} and it shows weak dependence on all remaining coefficients. A striking feature of its behavior is that $\langle P_{LT} \rangle$ changes its sign in the above-mentioned region of the new Wilson coefficients, while in the SM case its sign never changes. For this reason study of the magnitude and sign of $\langle P_{LT} \rangle$ can serve as a good test for looking new physics beyond the SM.

The dependence of $\langle P_{TL} \rangle$ on C_X for the $B \rightarrow \rho\mu^+\mu^-$ decay is presented in Fig. (5) in Region I and Fig. (6) in Region II, respectively. In both regions $\langle P_{TL} \rangle$ exhibits strong dependence on scalar C_{RLRL} and C_{LRRL} and tensor interaction coefficients. Moreover, when $C_{RLRL}(C_{LRRL})$ is negative (positive), $\langle P_{TL} \rangle$ is positive (negative). When $C_{TE} < -0.8(> 0)$ and $C_T < 0(> 2)$, $\langle P_{TL} \rangle$ is negative and positive otherwise. Hence determination of the magnitude and sign of $\langle P_{TL} \rangle$ gives unambiguous confirmation of the existence of new physics due to scalar and tensor interactions.

In Figs. (7) and (8) we present the dependence of $\langle P_{TT} \rangle$ on C_X for the $B \rightarrow \rho\mu^+\mu^-$ decay. In region I (see Fig. (7)) $\langle P_{TT} \rangle$ is strongly dependent on vector type interactions C_{LR} , C_{RR} and for the negative values of C_{LL} and C_T . On the other hand, in Region II, $\langle P_{TT} \rangle$ is strongly dependent only on tensor interaction. In Region I $\langle P_{TT} \rangle$ is positive (negative) for negative values of $C_{LR}(C_{RR})$ and it attains at negative (positive) values for positive values of $C_{LR}(C_{RR})$. In the second region the sign of $\langle P_{TT} \rangle$ changes only for the vector interaction C_{RR} .

Depicted in Figs. (9) and (10) are the dependence of $\langle P_{NN} \rangle$ on the new Wilson coefficients. The situation is quite similar to the previous case for the $\langle P_{TT} \rangle$. The only difference

being, $\langle P_{NN} \rangle$ in Region I depends strongly on C_{TE} rather than C_T , for their negative values, compared to that for the $\langle P_{TT} \rangle$ case.

All remaining double-lepton polarization asymmetries for the $B \rightarrow \rho\mu^+\mu^-$ decay are very small numerically and therefore we do not present them.

Through Figs. (11)–(14) we study the dependence of P_{ij} on the new Wilson coefficients for the $B \rightarrow \rho\tau^+\tau^-$ decay, which provides richer information about the new physics effects.

In Fig. (11) the dependence of $\langle P_{LL} \rangle$ on C_X is given. We observe from this figure that $\langle P_{LL} \rangle$ is very sensitive to all new Wilson coefficients except C_{RL} . It changes its sign only for the variations in C_T and for all rest of the new Wilson coefficients $\langle P_{LL} \rangle$ does not seem to change its sign. Therefore investigation of the sign of $\langle P_{LL} \rangle$ can give important clue about the existence of the tensor interaction.

In Fig. (12) we present the dependence of $\langle P_{LT} \rangle$ on the new Wilson coefficients. Noting that $\langle P_{TL} \rangle$ exhibits similar behavior, except several scalar coefficients, $\langle P_{LT} \rangle$ is sensitive to all remaining Wilson coefficients. Similar to the $\langle P_{LL} \rangle$ case, $\langle P_{LT} \rangle$ changes its sign in the presence of the tensor interaction and therefore this circumstance can be quite useful in looking for new physics beyond the SM.

The dependence of $\langle P_{LN} \rangle \approx -\langle P_{NL} \rangle$ on C_X is presented in Fig. (13). We see from this figure that $\langle P_{LN} \rangle$ is very sensitive to all new Wilson coefficients, especially to the vector interaction coefficients C_{LL} and C_{LR} .

In Fig. (14) we present the dependence of $\langle P_{NN} \rangle \approx -\langle P_{TT} \rangle$ on the new Wilson coefficients. We observe from this figure that when C_X is negative $\langle P_{NN} \rangle > \langle P_{NN}^{SM} \rangle$ for the coefficients C_{LR} , C_{LL} , C_{LRRL} and C_T , and $\langle P_{NN} \rangle > \langle P_{NN}^{SM} \rangle$ for the coefficients C_{RL} , C_{RR} , C_{RLLR} and C_{TE} . On the other hand, when C_X is positive the situation changes to the contrary, except for the tensor interaction (neglecting the narrow region for the coefficient C_{TE}). The numerical analysis for the rest of the remaining double-lepton polarization asymmetries for the $B \rightarrow \rho\tau^+\tau^-$ decay are not presented in this work due to their negligible smallness.

It follows from the present analysis that few of the double-lepton polarization asymmetries show considerable departure from the SM predictions and these ones are strongly dependent on different types of interactions. Hence, the study of these quantities can play crucial role in establishing new physics beyond the SM.

At the end of this section, we would like to discuss the following problem. Could there be a case in which the branching ratio coincides with that of the SM result, while double-lepton polarization asymmetry does not? In order to answer this question we study the correlation between the $\langle P_{ij} \rangle$ and the branching ratio \mathcal{B} . We can briefly summarize the results of our numerical analysis as follows: For the $B \rightarrow \rho\mu^+\mu^-$ decay, except for a very narrow region of C_{RR} , such a region is absent for all new Wilson coefficients for all of the asymmetries $\langle P_{ij} \rangle$.

The $B \rightarrow \rho\tau^+\tau^-$ decay is more informative for this aim, which are measurable in the experiments. In Figs. (15) and (16) we present the dependence of $\langle P_{LL} \rangle$ and $\langle P_{LT} \rangle$ on the branching ratio. It follows from these figures that, there indeed exists certain regions of C_X for which the double-lepton polarization asymmetry differs from the SM prediction, while the branching ratio coincides with that of the SM result. We also note that, such a region exists for the remaining double-lepton polarization asymmetries for the tensor interaction

as well.

In conclusion, in the present work we investigate the double-lepton polarization asymmetries when both leptons are polarized, using a general, model independent form of the effective Hamiltonian. We obtain that various double-lepton polarization asymmetries can serve as a good test in looking for new physics beyond the SM. We also study the correlation between $\langle P_{ij} \rangle$ and the branching ratio for the $B \rightarrow \rho \tau^+ \tau^-$ decay and find out that there exist regions of the new Wilson coefficients for which the double-lepton polarization asymmetry differs considerably from the SM prediction, while the branching ratio coincides with the SM prediction. Therefore in these regions the new physics effects can be established just by measuring the double-lepton polarizations.

References

- [1] T. M. Aliev, M. K. Çakmak and M. Savcı, Nucl. Phys. B **607**, 305 (2001); T. M. Aliev and M. Savcı, Phys. Lett. B **481**, 275 (2000).
- [2] S. Rai Choudhury, A. Gupta and N. Gaur, Phys. Rev. D **60**, 115004 (1999); S. Fukae, C. S. Kim and T. Yoshikawa, Phys. Rev. D **61**, 074015 (2000); T. M. Aliev, M. K. Çakmak, A. Özpıneci and M. Savcı, Phys. Rev. D **64**, 055007 (2001); D. Guetta and E. Nardi, Phys. Rev. D **58**, 012001 (1998).
- [3] F. Krüger and L. M. Sehgal, Phys. Lett. B **380**, 199 (1996); J. L. Hewett, Phys. Rev. D **53**, 4964 (1996).
- [4] T. M. Aliev, M. K. Çakmak, A. Özpıneci and M. Savcı, Phys. Rev. D **64**, 055007 (2001); T. M. Aliev, M. Savcı, A. Özpıneci and H. Koru, J. Phys. G **24**, 49 (1998).
- [5] S. R. Choudhury and N. Gaur, Phys. Rev. D **66**, 094015 (2002).
- [6] S. Rai Choudhury, N. Gaur and N. Mahajan, Phys. Rev. D **66**, 054003 (2002); E. O. İltan and G. Turan, Phys. Rev. D **61**, 034010 (2000).
- [7] T. M. Aliev, A. Özpıneci, M. Savcı, Phys. Lett. B **520**, 69 (2001).
- [8] W. Bensalem, D. London, N. Sinha, and R. Sinha, Phys. Rev. D **67**, 034007 (2003).
- [9] S. R. Choudhury, N. Gaur, A. S. Cornell and G. C. Joshi, *Phys. Rev.* **D 68** (2003) 054016.
- [10] F. Krüger, L. Sehgal, *Phys. Rev.* **D 56** (1997) 5432; *Phys. Rev.* **D 60** (1999) 099905(E).
- [11] E. O. İltan, *Int. J. Mod. Phys.* **A 14** (1999) 4365;
T. M. Aliev and M. Savcı, *Phys. Rev.* **D 60** (1999) 014005;
S. R. Choudhury and N. Gaur, *Phys. Rev.* **D 66** (2002) 094015.
- [12] T. M. Aliev, V. Bashiry, M. Savcı, prep:hep-ph/0504215 (2005).
- [13] T. M. Aliev, V. Bashiry, M. Savcı, *Eur. Phys. J.* **35** (2004) 197.
- [14] T. M. Aliev, V. Bashiry, M. Savcı, *Phys. Rev.* **D 71** (2005) 035013.
- [15] M. Jezabek and J. H. Kühn, *Nucl. Phys.* **B 320** (1989) 20.
- [16] M. Misiak, *Nucl. Phys.* **B 439** (1995) 461.
- [17] G. Buchalla, G. Isidori and S. J. Rey, *Nucl. Phys.* **B 511** (1996) 594.
- [18] A. Ali, T. Mannel and T. Morozumi, *Phys. Lett.* **B 273** (1991) 505.
- [19] Z. Ligeti, I. W. Stewart and M. B. Wise, *Phys. Rev.* **D 53** (1996) 4987.
- [20] F. Krüger and L. Sehgal, *Phys. Rev.* **D 53** (1996) 4937.

- [21] Z. Ligeti and M. B. Wise, *Phys. Lett. B* **420** (1998) 359.
- [22] C. Q. Geng and C. P. Kao, *Phys. Rev. D* **57** (1998) 4479.
- [23] T. M. Aliev, C. S. Kim and Y. G. Kim, *Phys. Rev. D* **62** (2000) 014026.
- [24] P. Ball, R. Zwicky, *Phys. Rev. D* **71** (2005) 014029.
- [25] A. Ishikawa et al., Belle Collaboration, *Phys. Rev. Lett.* **91** (2003) 261601.
- [26] B. Aubert et al., BaBar Collaboration, *Phys. Rev. Lett.* **91** (2003) 221802.
- [27] V. M. Abazov et al., DO Collaboration, *Phys. Rev. Lett.* **94** (2005) 071802.

Figure captions

Fig. (1) The dependence of the averaged double-lepton polarization asymmetry $\langle P_{LL} \rangle$ on the new Wilson coefficients C_X , for the $B \rightarrow \rho\mu^+\mu^-$ decay, in Region I.

Fig. (2) The same as in Fig. (1), but in Region II.

Fig. (3) The same as in Fig. (1), but for the averaged double-lepton polarization asymmetry $\langle P_{LT} \rangle$.

Fig. (4) The same as in Fig. (3), but in Region II.

Fig. (5) The same as in Fig. (1), but for the averaged double-lepton polarization asymmetry $\langle P_{TL} \rangle$.

Fig. (6) The same as in Fig. (5), but in Region II.

Fig. (7) The same as in Fig. (1), but for the averaged double-lepton polarization asymmetry $\langle P_{TT} \rangle$.

Fig. (8) The same as in Fig. (7), but in Region II.

Fig. (9) The same as in Fig. (1), but for the averaged double-lepton polarization asymmetry $\langle P_{NN} \rangle$.

Fig. (10) The same as in Fig. (9), but in Region II.

Fig. (11) The dependence of the averaged double-lepton polarization asymmetry $\langle P_{LL} \rangle$ on the new Wilson coefficients C_X , for the $B \rightarrow \rho\tau^+\tau^-$ decay, in Region II.

Fig. (12) The same as in Fig. (11), but for the $\langle P_{LT} \rangle$.

Fig. (13) The same as in Fig. (11), but for the $\langle P_{LN} \rangle$.

Fig. (14) The same as in Fig. (11), but for the $\langle P_{NN} \rangle$.

Fig. (15) Parametric plot of the correlation between the averaged double-lepton polarization asymmetry $\langle P_{LL} \rangle$ and the branching ratio for the $B \rightarrow \rho\tau^+\tau^-$ decay, in Region II.

Fig. (16) Parametric plot of the correlation between the averaged double-lepton polarization asymmetry $\langle P_{LT} \rangle$ and the branching ratio for the $B \rightarrow \rho\tau^+\tau^-$ decay, in Region II.

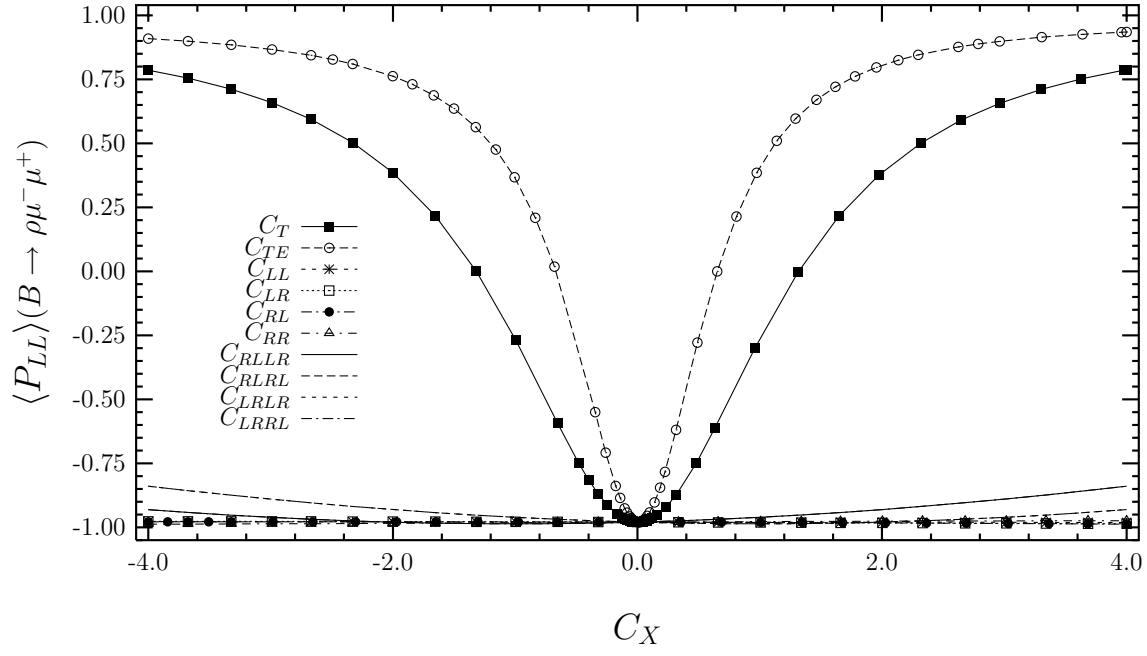


Figure 1:

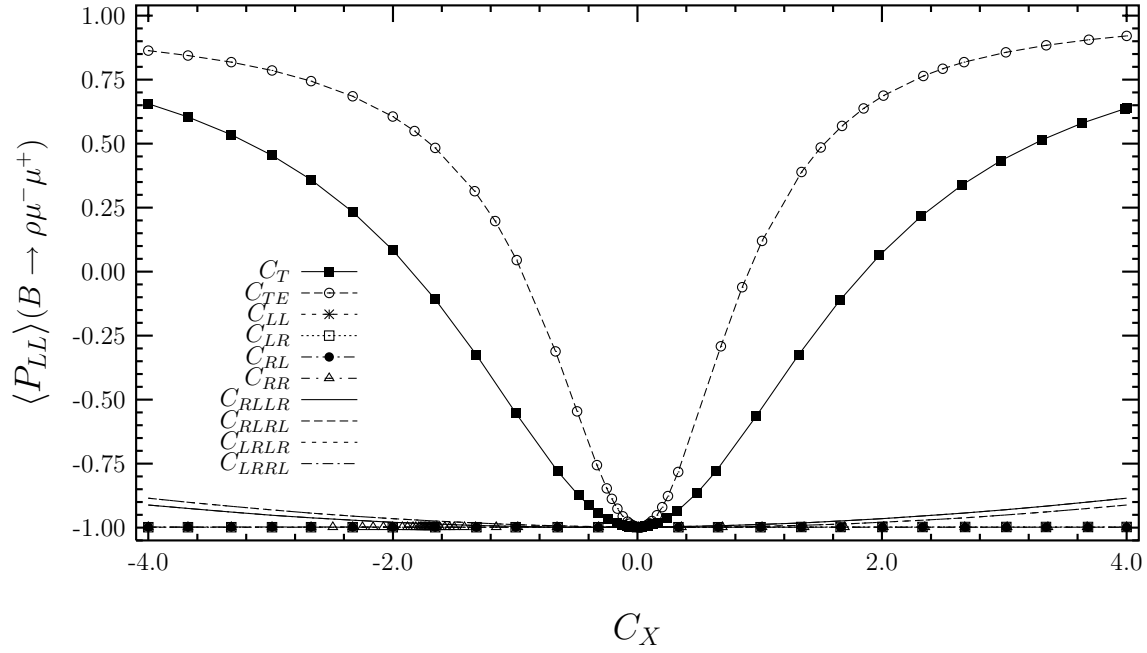


Figure 2:

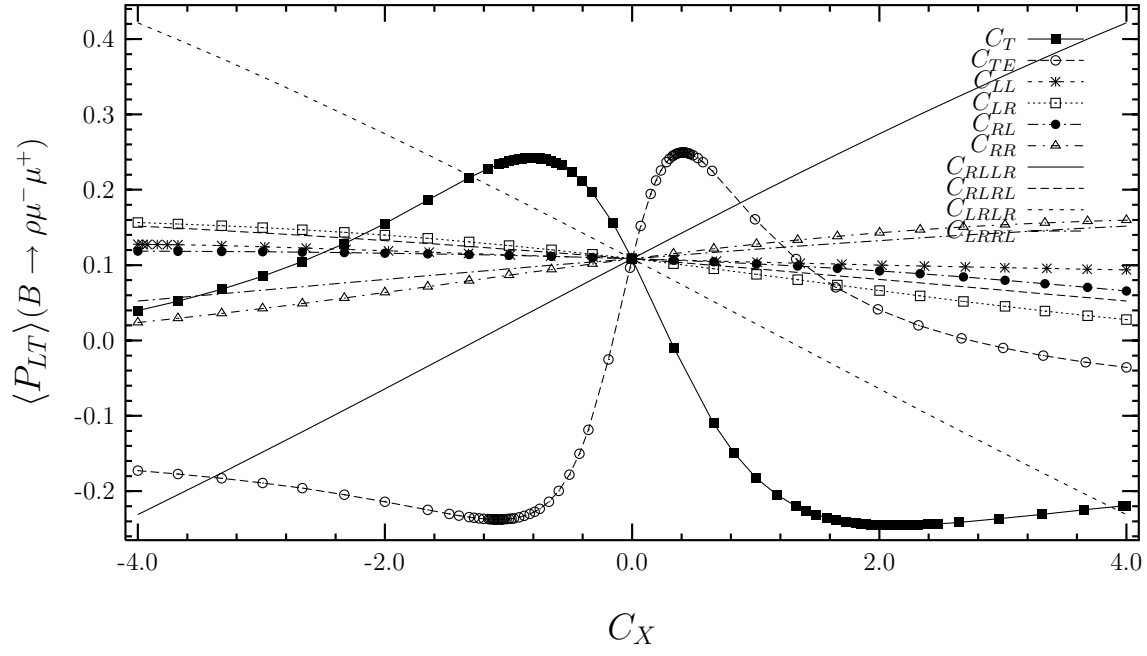


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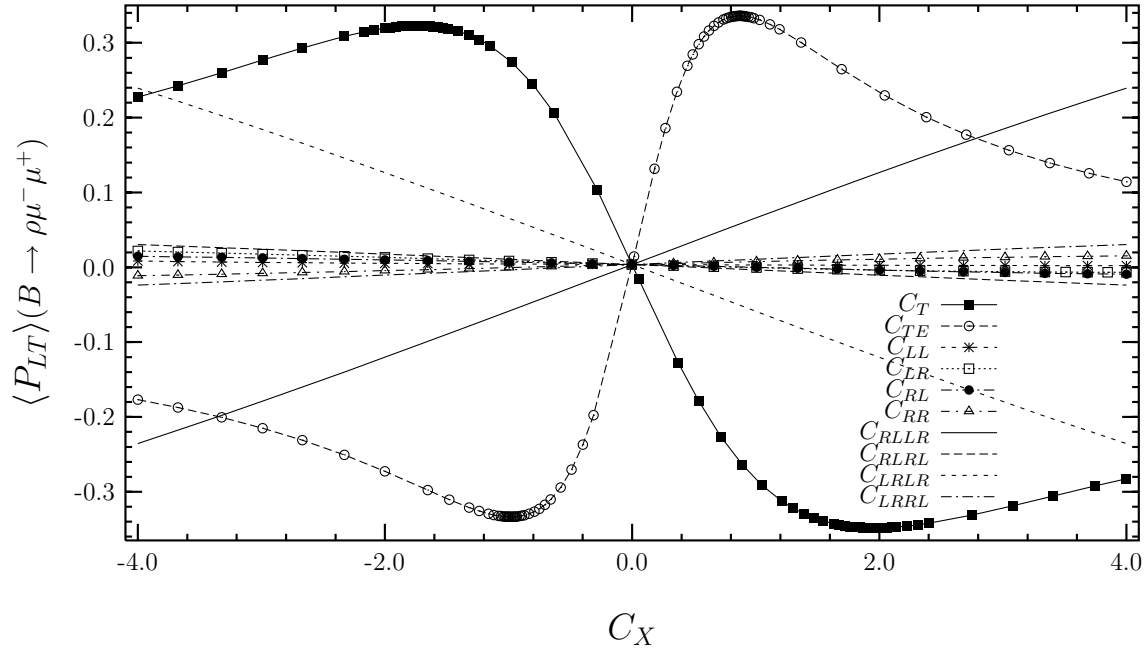


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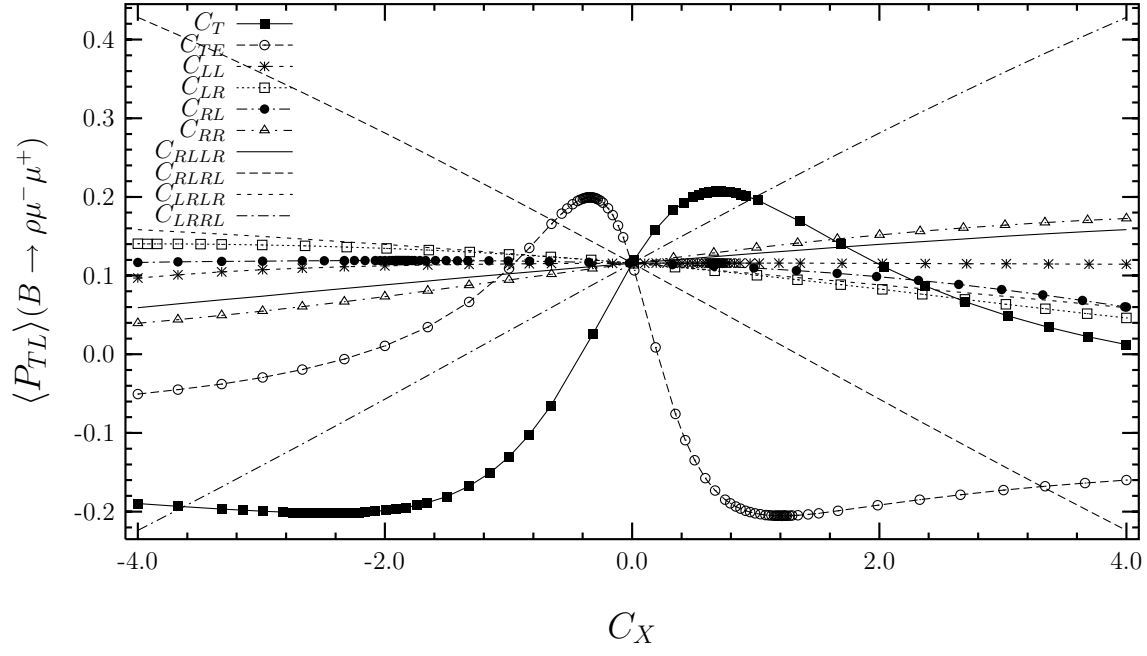


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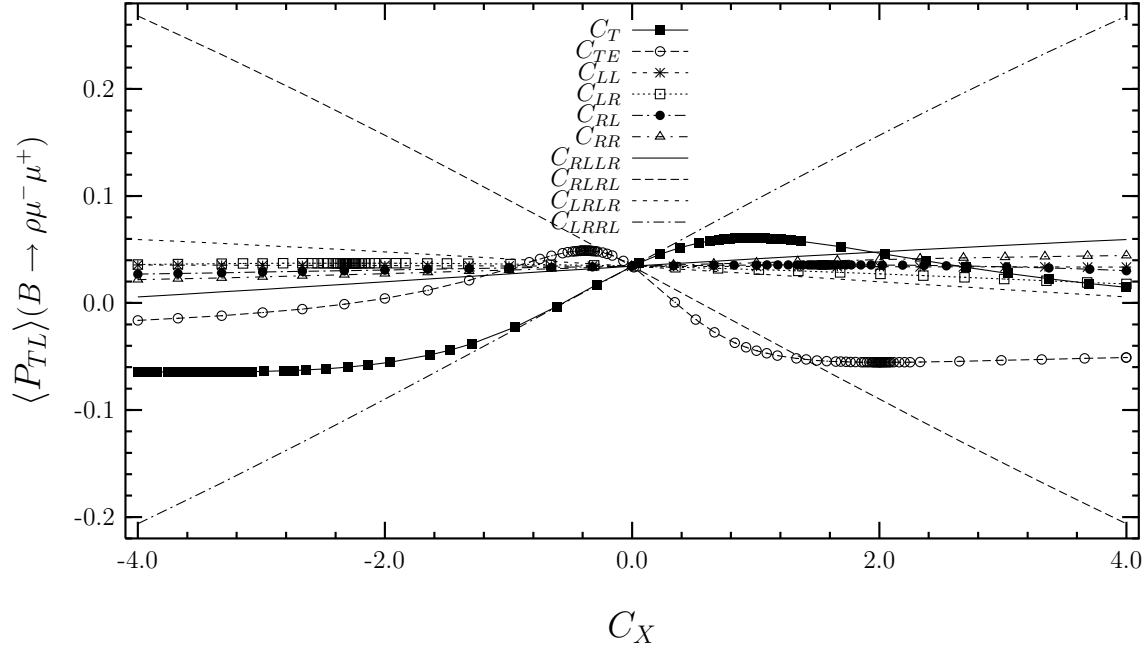


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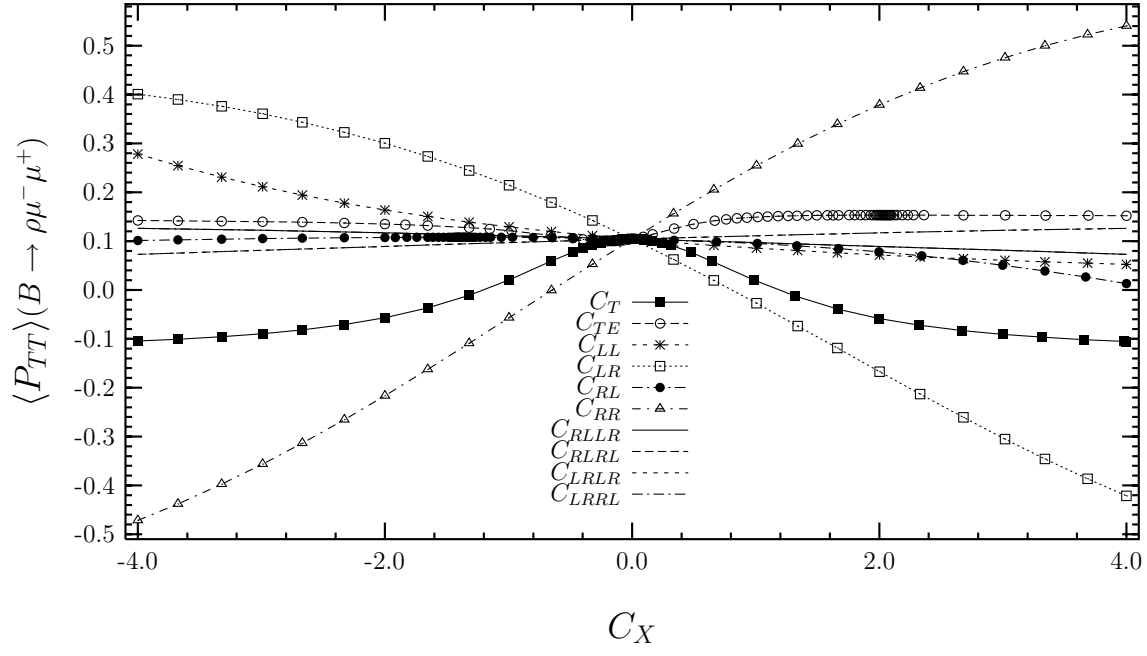


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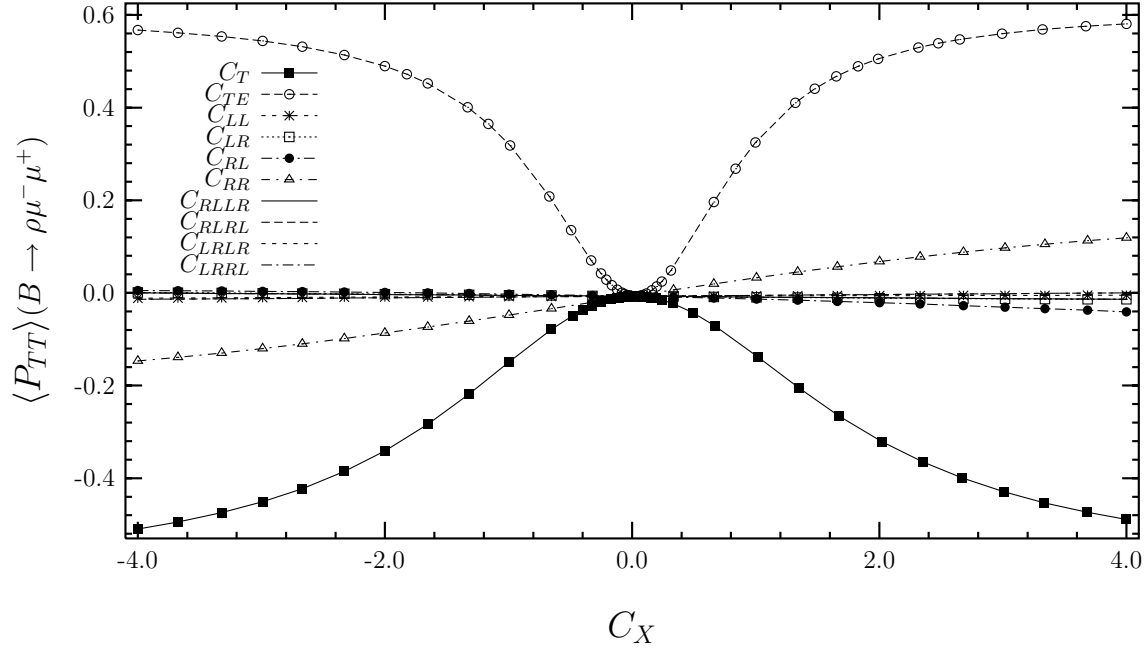


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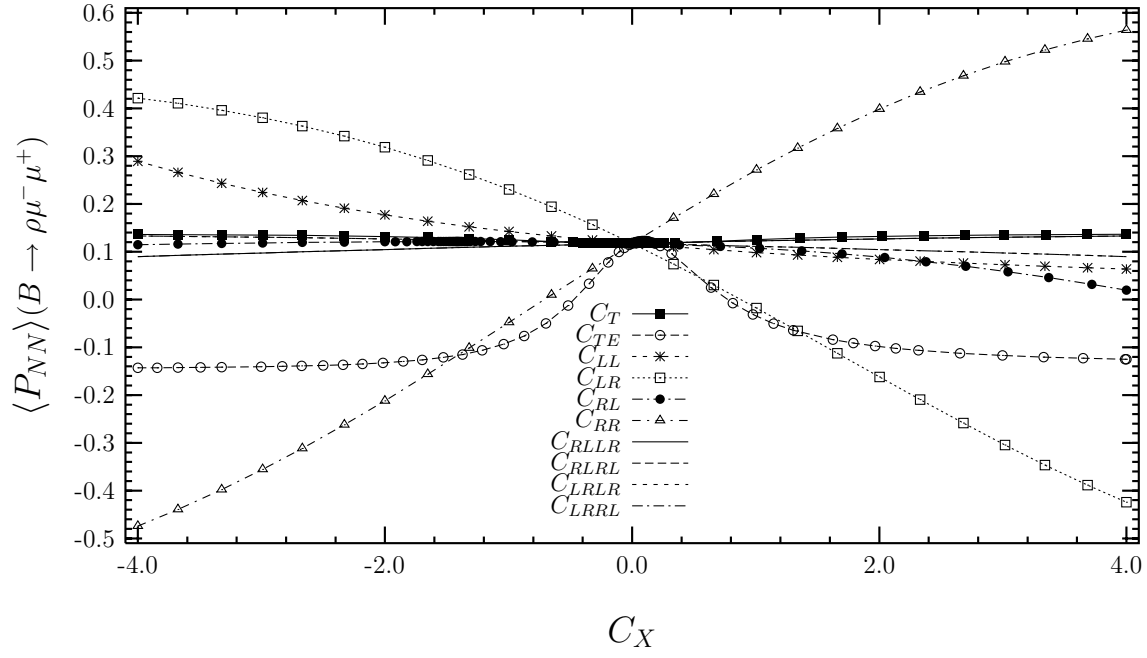


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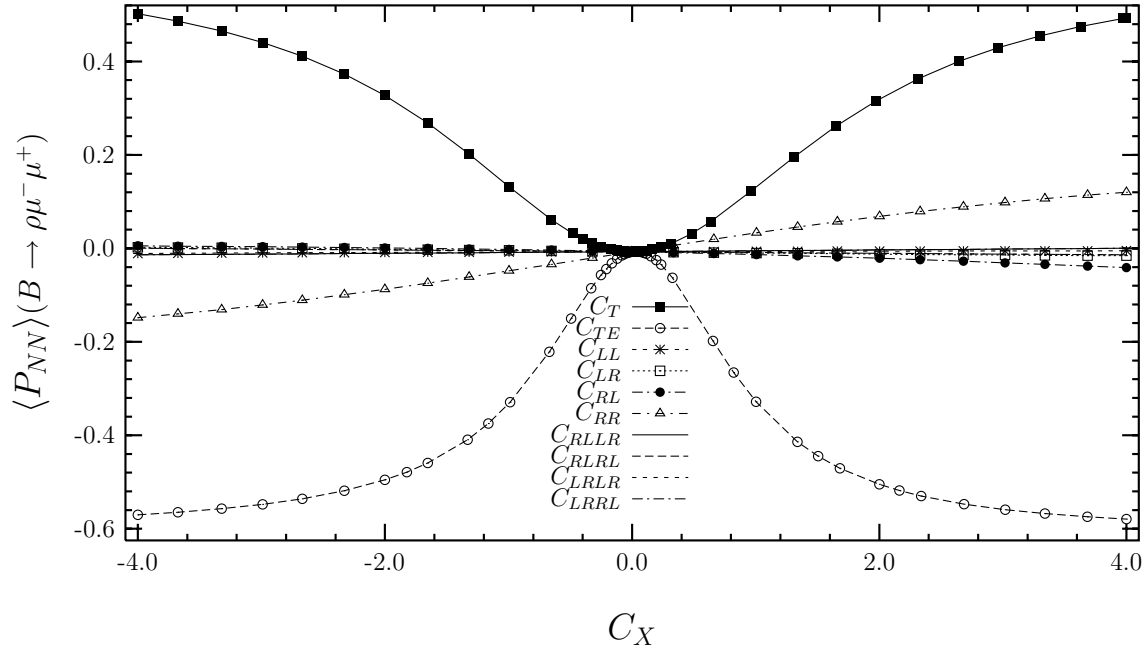


Figure 10:

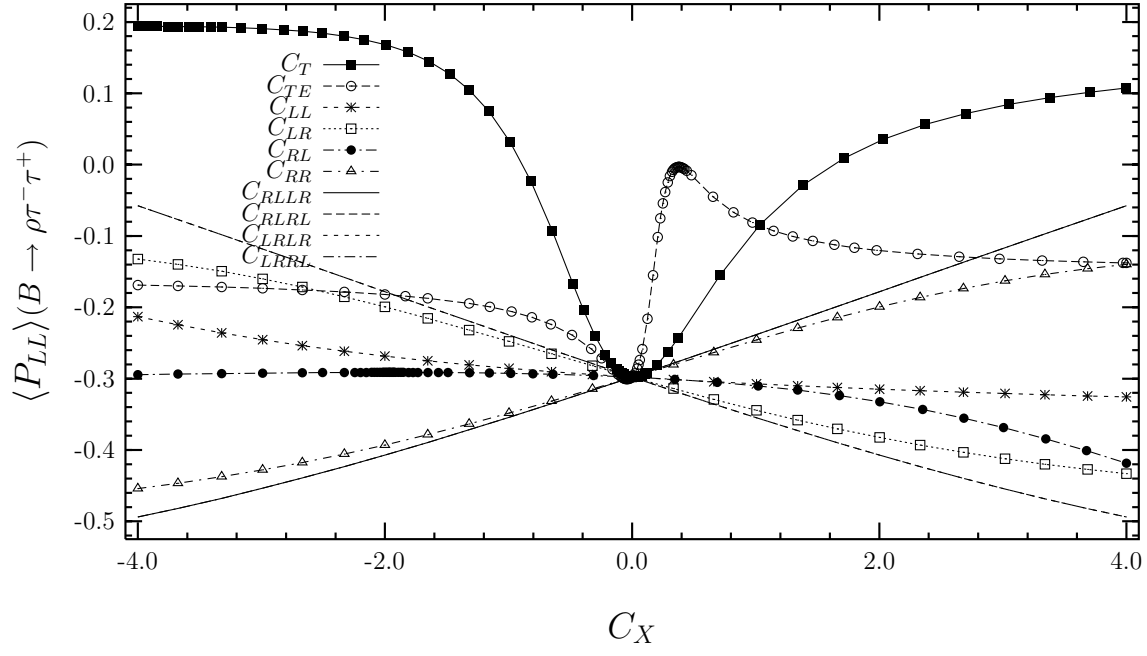


Figure 11:

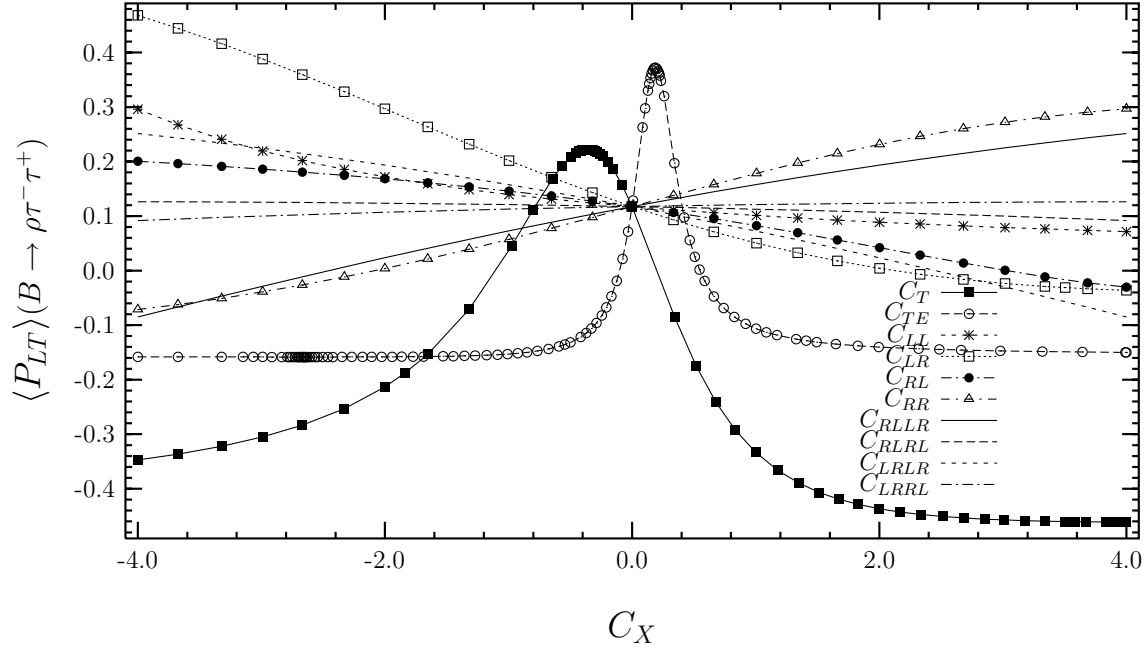


Figure 12:

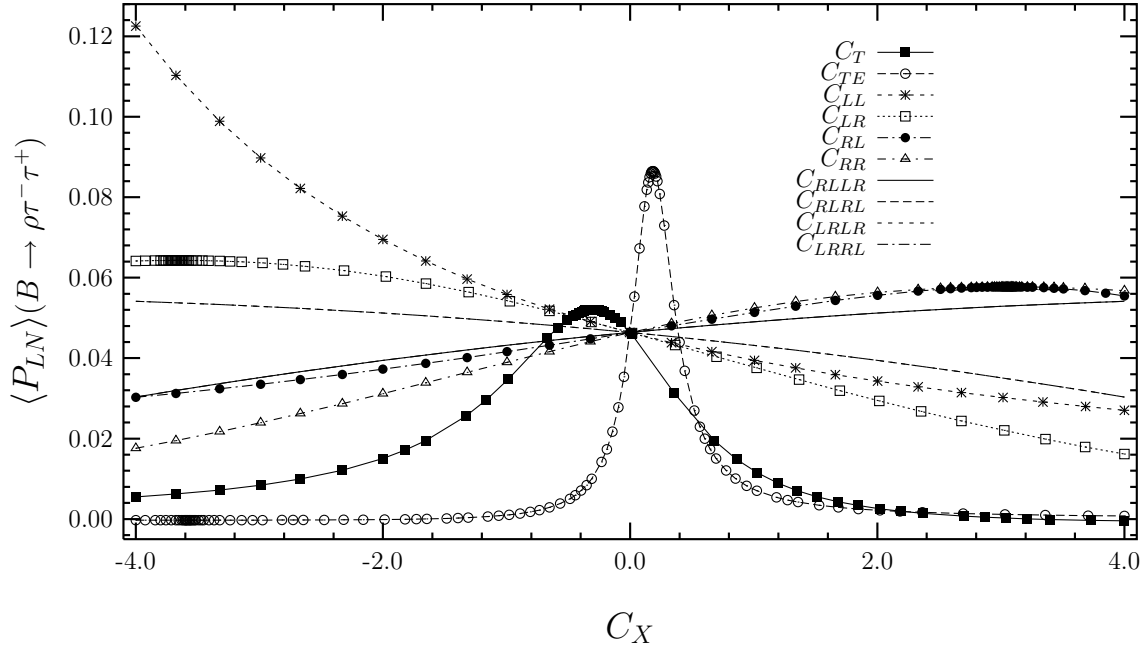


Figure 13:

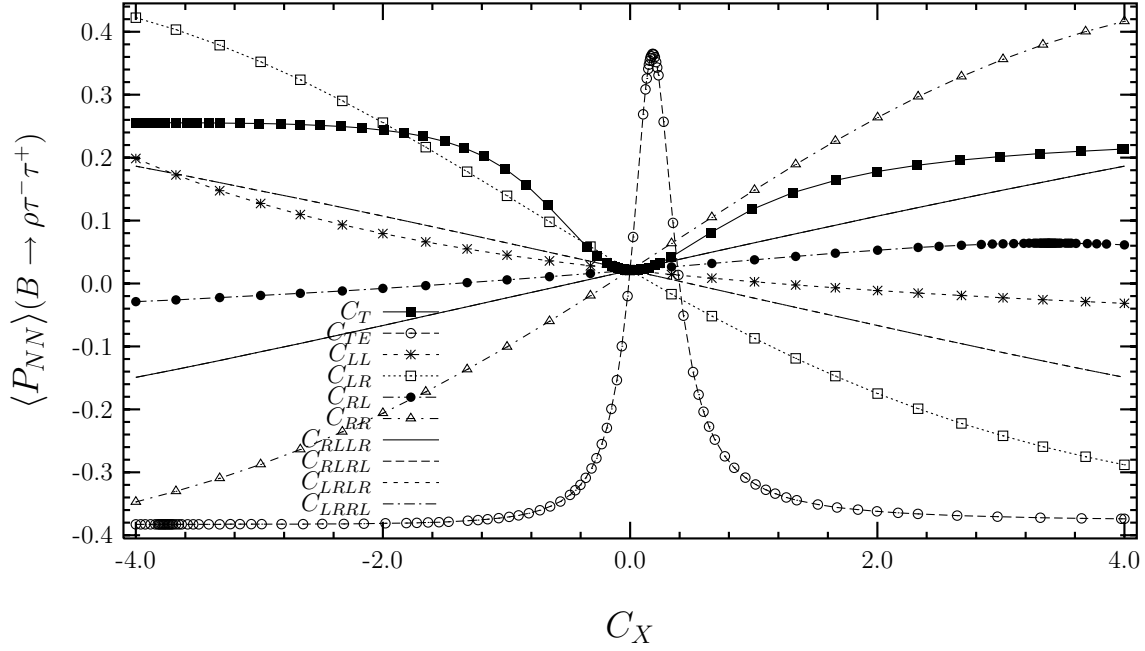


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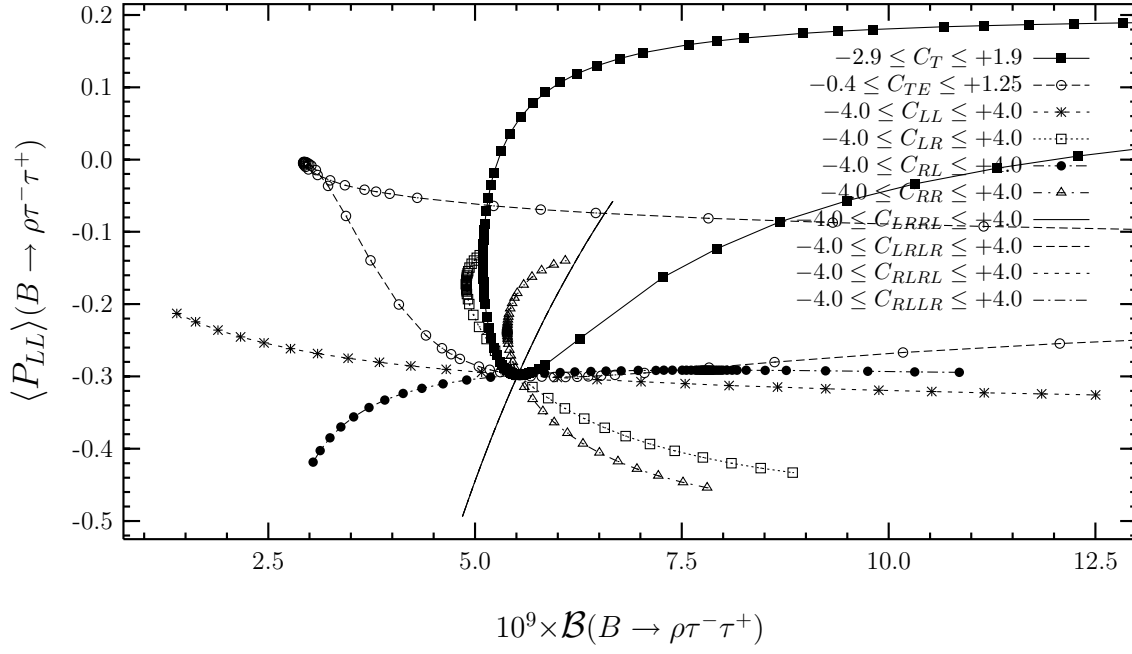


Figure 15:

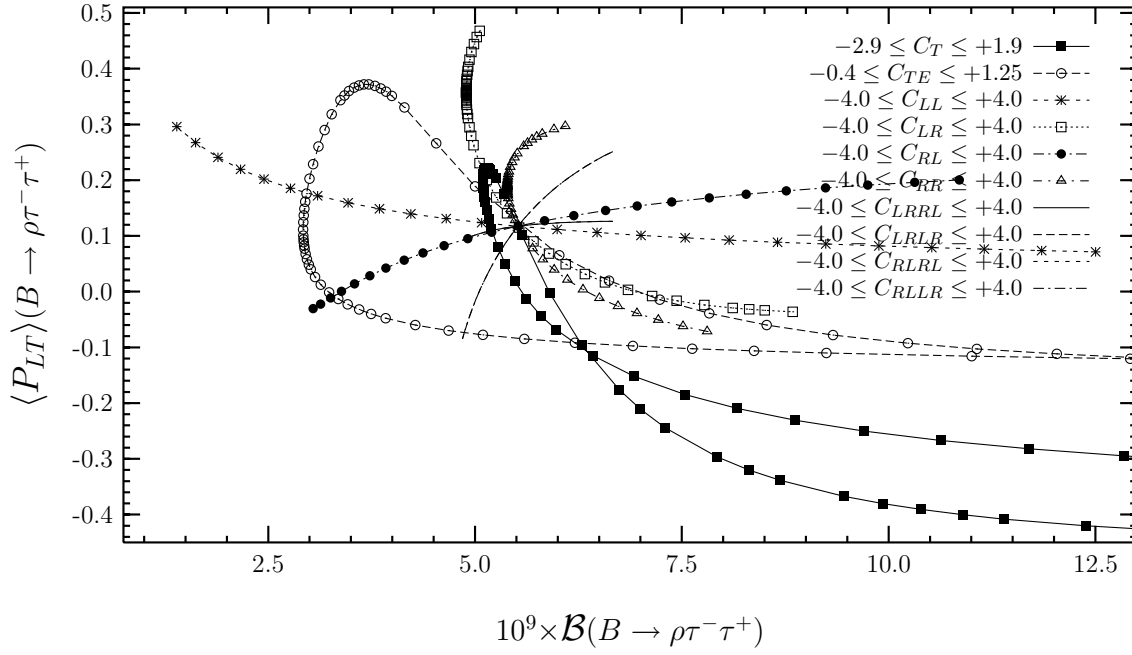


Figure 16: